UNIVERSITY OF CAMBRIDGE

Model Predictive Control on an FPGA: *Aerospace and Space Scenarios*

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Monday 8th September 2014: 14:00–15:00

Cambridge University Engineering Department

Introduction

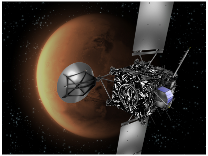
1. What is MPC? *(You probably already know)*
   What is an FPGA?

2. Interior point QP-based MPC on an FPGA, for aircraft control

3. Interior point LP-based VH-LTV-MPC on an FPGA, for medium-range spacecraft rendezvous

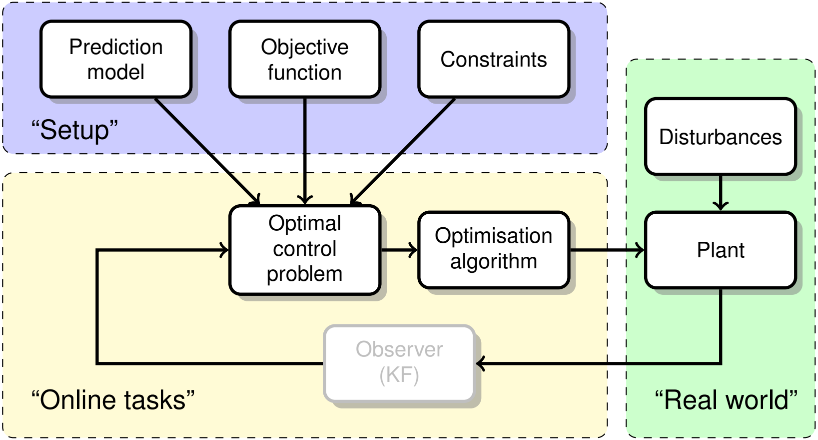
4. First-order QP-based MPC on an FPGA, for terminal spacecraft rendezvous

5. Conclusions and lessons learnt



Introduction

Model Predictive Control



Introduction

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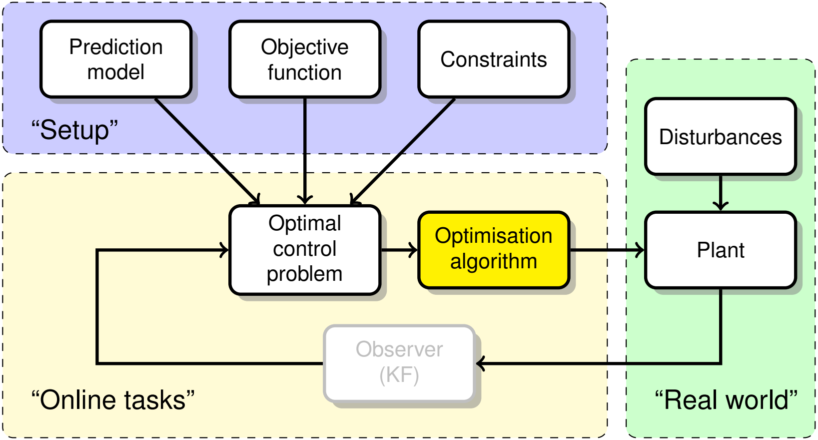
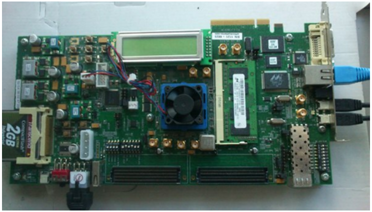


Introduction

Field Programmable Gate Array

- **F**ield **P**rogrammable **G**ate **A**rray
  - Programmable hardware
  - Contains many logic blocks:
    Lookup tables, Flip-flops, RAM, Dedicated Multipliers
  - User specifies how these should be connected together

- Implementing a circuit for an algorithm on an FPGA is not like programming a microprocessor
  - Multiple clocks
  - Parallelism
  - Timing
  - Custom numerical representations



Introduction

. . . and what's this got to do with MPC?

Raw speed

- Exploit parallelism
- Make the controller latency ever lower
- Control fast processes

Latency vs. clock rate

- Achieve controller latency similar to running MPC on a desktop PC, but on embedded hardware at much lower clock rates

Embeddability

- System on a chip
- Power consumption advantages?

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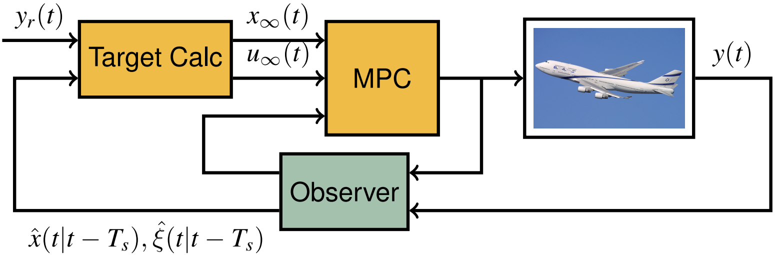


Control Objective

- MPC: Control roll and pitch of B747
- Use all actuators and all state measurements
- Outer loop provides roll/pitch setpoints to track altitude/yaw trajectory

Aerospace Application

Plant description

MIMO aircraft control

- 12 **states:**
  - Roll rate, Pitch rate, Yaw Rate
  - Airspeed, AoA, Sideslip
  - Roll, Pitch
  - 4 engine dynamics (1st order lag)
- 17 **inputs:**
  - 4 elevators, 1 trimmable horizontal stabiliser
  - 4 ailerons, 2 spoiler banks
  - 2 rudders
  - 4 engine setpoints
- 34 **contraints:** Upper/lower bound on each input

Why MPC

- Multivariable control
- Contraints
- Reconfiguration (not addressed here)

... on an FPGA?

- Embeddability
- Latency vs clock rate

MPC parameters

- Quadratic cost
- Tracking
- No state constraints
- Prediction horizon $N = 5$ or $N = 12$
- Uncondensed formulation (sparse with equality constraints)

Finite horizon OCP

$$\min_{\theta} \|\delta x_N - \delta x_s\|_P^2 + \sum_{i=0}^{N-1} \left(\|\delta x_i - \delta x_s\|_Q^2 + \|\delta u_i - \delta u_s\|_R^2 \right) \quad \text{(1a)}$$

$$\text{subject to:} \quad \delta x_0 = \delta \hat{x}(k) \quad \text{(1b)}$$

$$\delta x_{i+1} = A\delta x_i + B\delta u_i + B_d \hat{w}(k), \; i = 0, \ldots, N-1 \quad \text{(1c)}$$

$$\delta u_{\min} \leq \delta u_i \leq \delta u_{\max}, \quad i = 0, \ldots, N-1. \quad \text{(1d)}$$

- (The δ indicates deviation from trim point used for linearisation)
- $\| \cdot \|_X^2 \triangleq \cdot^T X \cdot$

OCP as a QP

$$\min_{\theta} \frac{1}{2}\theta^T H\theta + h^T\theta \quad \text{subject to} \quad G\theta \leq g,\ F\theta = f.$$

where...

$$H \triangleq 2\left(I_N \otimes (Q \oplus R)\right) \oplus P$$

$$G \triangleq \left[I_N \otimes \begin{bmatrix} 0_{m\times n} & I_m \\ 0_{m\times n} & -I_m \end{bmatrix},\ 0_{2Nm\times n} \right]$$

$$F \triangleq \begin{bmatrix} -I_n & & & & \\ A & B & -I_n & & \\ & & \vdots & \ddots & \\ & & & & -I_n \end{bmatrix}$$

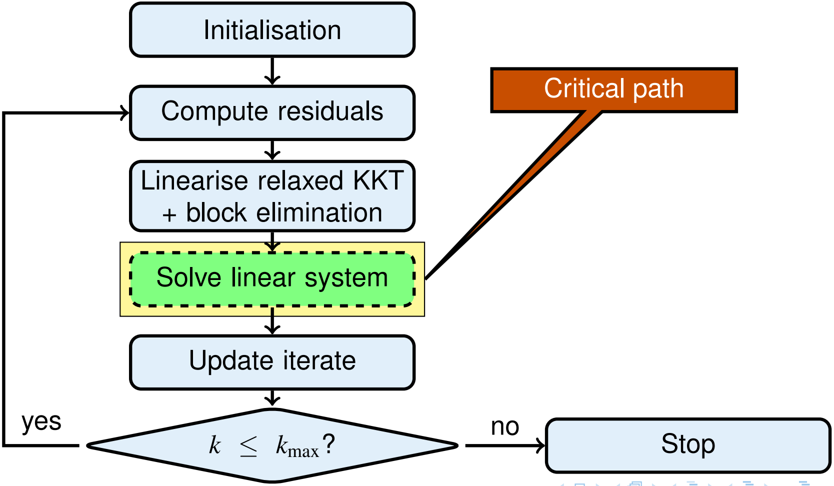
$$h \triangleq \begin{bmatrix} -\mathbf{1}_N^T \otimes \begin{bmatrix} \delta x_\infty^T Q & \delta u_\infty^T R \end{bmatrix} & -\delta x_\infty^T P \end{bmatrix}^T$$

$$g \triangleq \mathbf{1}_N \otimes \begin{bmatrix} \delta u_{\max}^T & -\delta u_{\min}^T \end{bmatrix}^T$$

- $\oplus$: Direct sum
- $\otimes$: Kronecker product

Aerospace Application

Primal-Dual Interior Point Method



Aerospace Application

Solving the Linear System

Computational bottleneck

$$\underbrace{\begin{bmatrix} H + \Phi_k & F^T \\ F & 0 \end{bmatrix}}_{\mathcal{A}_k} \begin{bmatrix} \Delta\theta_k \\ \Delta\nu_k \end{bmatrix} = b_k$$

Conventional approach

- Factorise + substitute
- ☹ Many divisions
- ☹ Cannot terminate early
- ☹ Difficult to parallelise

Alternative approach applied

- Solve using iterative MINRES (Minimum Residual) algorithm

- ☺ Iterations vs accuracy
- ☺ Matrix-vector multiplication: very parallelisable
- ☺ Sparse structure (and most elements do not change)

- ☹ Sensitive to conditioning
- ☹ Sensitive to precision
- ☹ Inefficient without parallelisation

Aerospace Application

Problem Scaling

- Linear system conditioning important
- Diagonal online preconditioner
- **Offline model scaling**: Consider the substitution

$$
\begin{aligned}
A &\leftarrow T_Q A T_Q^{-1} & \qquad B &\leftarrow T_Q B T_R^{-1} \\
Q &\leftarrow T_Q^{-1} Q T_Q^{-1} & \qquad R &\leftarrow T_R^{-1} R T_R^{-1} \\
P &\leftarrow T_Q^{-1} P T_Q^{-1} & &
\end{aligned}
$$

for diagonal $T_Q > 0$ and $T_R > 0$, and corresponding scalings on the constraints.

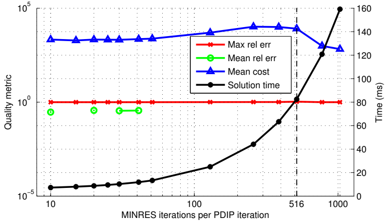
Choose T_Q, T_R to approximately normalise the 2-norms of the rows of

$$
\begin{bmatrix} H + \cancel{\Phi_k} & F^T \\ F & 0 \end{bmatrix}
$$

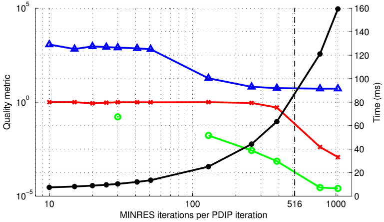
(Heuristic, but effective)

Aerospace Application

Problem Scaling: importance



(a) No preconditioning

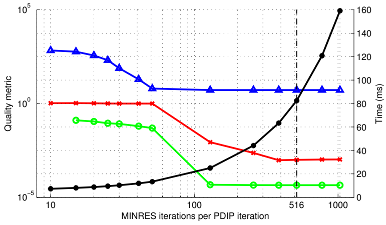


(b) Online preconditioning only

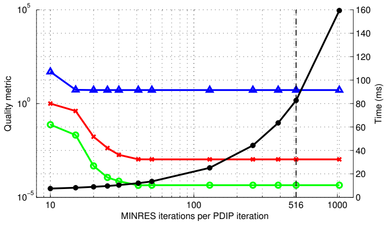
- Based on closed-loop simulation in software
- Timing estimate for $250\ \mathrm{MHz}$ FPGA based on analytical formula
- Without preconditioning, MINRES-based solver is disastrous
- With online preconditioning good results if enough MINRES iterations per PDIP iteration

Aerospace Application

Problem Scaling: importance



(c) Offline preconditioning only



(d) Online & Offline Preconditioning

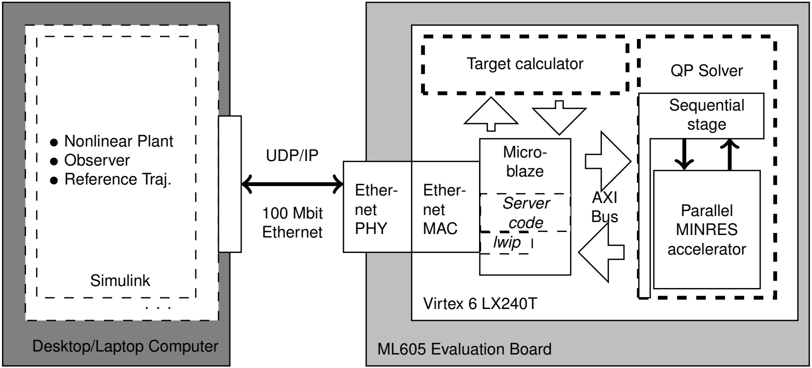
- With offline preconditioning, solution quality for low MINRES iteration counts improves
- Best results with both offline and online preconditioning

General Implementation

- Pure VHDL solver coded by Juan Jerez (Imperial, now ETH).
- Prediction model hard coded in ROM.
- Connect to MicroBlaze for HIL setup
- Software server on MicroBlaze enables communication over ethernet with plant model in Simulink, using UDP/IP

Aerospace Application

Hardware-in-the-loop Setup



Aerospace Application

Computation time comparison

Implementation				Relative numerical accuracy		Mean	Max Solution time	
QP Solver	Bits	N	I_{MR}	$e_{\max}$	e_{μ}	cost	QP (ms)	Clock cycles
F /P-MINRES	32	12	51	9.67×10^{-4}	3.02×10^{-5}	5.2246	12	2.89×10^{6}
PC/RWR1998	64	12	–	–	–	5.2247	23	5.59×10^{7}
PC/FORCES	64	12	–	5.89×10^{-3}	1.69×10^{-4}	5.2250	13	3.09×10^{7}
UB/FORCES	32	12	–	3.83×10^{-3}	7.31×10^{-5}	5.2249	1911	1.91×10^{8}
F /P-MINRES	32	5	30	9.10×10^{-4}	2.95×10^{-5}	5.2203	4	1.09×10^{6}
PC/RWR1998	64	5	–	–	–	5.2204	11	2.64×10^{7}
PC/CVXGEN	64	5	–	1.04×10^{-3}	1.84×10^{-5}	5.2203	3	7.20×10^{6}
PC/FORCES	64	5	–	5.00×10^{-3}	1.24×10^{-4}	5.2207	6	1.44×10^{7}
UB/CVXGEN	32	5	–	??	??	??	(269)	(2.69×10^{7})
UB/FORCES	32	5	–	4.14×10^{-3}	8.01×10^{-5}	5.2205	823	8.23×10^{7}

(FPGA QP solver (F) running at 250 MHz, *PC (PC) at* 2.4 GHz *and MicroBlaze (UB) at* 100 MHz. *(–) indicates a baseline. (??) indicates that meaningful data for control could not be obtained). P-MINRES indicates preconditioned MINRES. RWR1998 indicates Rao-Wright-Rawlings-Riccati-Recursion.*

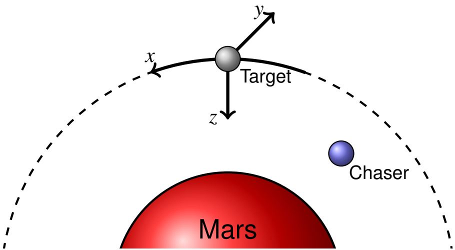
E. N. Hartley, J. L. Jerez, A. Suardi, J. M. Maciejowski, E. C. Kerrigan, and G. A. Constantinides.
Predictive control of a Boeing 747 aircraft using an FPGA.
In *Proceedings of the IFAC conference on Nonlinear Model Predictive Control*, August 23–27 2012.
doi: 10.3182/20120823-5-NL-3013.00016

E. N. Hartley, J. L. Jerez, A. Suardi, J. M. Maciejowski, E. C. Kerrigan, and G. A. Constantinides.
Predictive control using an FPGA with application to aircraft control.
IEEE Transactions on Control Systems Technology, 22(3):1006–1017, 2014.
doi: 10.1109/TCST.2013.2271791

Spacecraft Application I

Motivation

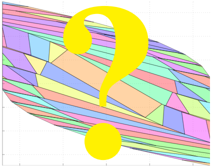
Rendezvous and Capture Scenario (Not to scale!)



- Active control of “chaser” to capture passive target
- Use fuel efficiently
- Cope with parametric and navigation uncertainty
- Work in circular or elliptical orbit

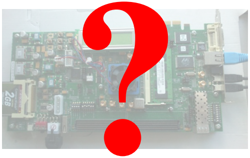
Spacecraft Application I

Why MPC on an FPGA?



Why MPC?

- Optimisation in the loop
- Natural constraint handling
- Potential for improved autonomy?



Why on an FPGA?

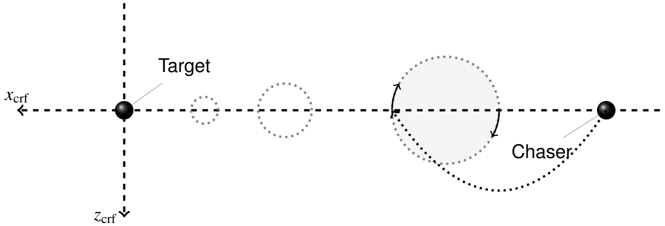
- MPC more computationally demanding (online) than PID, LQR, H_∞.
- Power, solar radiation
- Clock frequency vs. parallelism vs. numerical precision vs. resource usage
- (Cycle-accurate timing)

Spacecraft Application I

Problem Description

Control objective

- Bring chaser spacecraft from 10 km to 100 m in a fuel efficient manner
- Stop at a sequence of “holding points” reducing separation from target
  - (These are periodic trajectories centred at prescribed separations.)



MPC Formulation to reach next holding point

- Elliptical orbit $\Longrightarrow$ LTV prediction model *(Yamanaka-Ankersen)*
- Fuel minimisation proportional to thrust $\Longrightarrow$ $1-$norm cost
- Completion-type problem $\Longrightarrow$ variable horizon

$$J^* = \min_{N,\theta} \sum_{i=0}^{N-1} (1 + \gamma \|u_i\|_1) \quad \text{(2a)}$$

$$\begin{aligned} \text{s.t.} \quad x_0 &= x(k), && \text{(2b)} \\ x_{i+1} &= A_i x_i + B_i u_i, \quad i \in \{0, \ldots, N-1\}, && \text{(2c)} \\ x_N &= x_T(k+N), && \text{(2d)} \\ 0 \leq u_i &\leq u_{\max}, \; i \in \{0, \ldots, N-1\}, && \text{(2e)} \\ N &\leq N_{\max}, && \text{(2f)} \end{aligned}$$

Spacecraft Application I

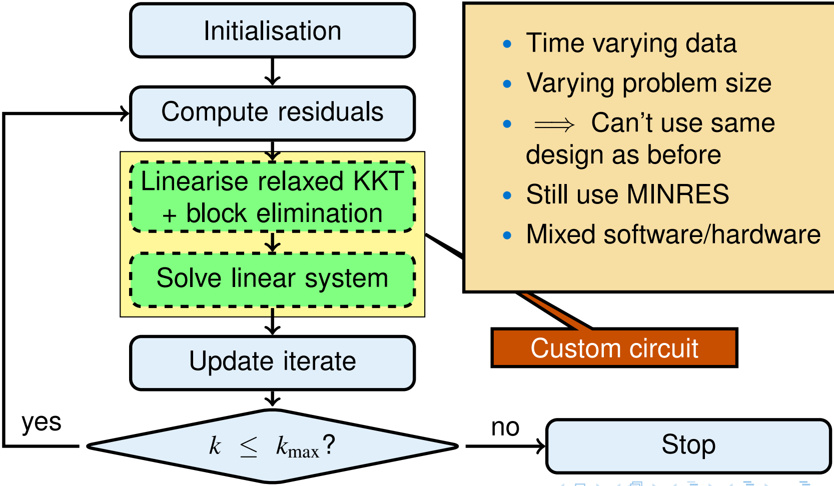
Convexification of variable horizon problem

Variable horizon MPC controller

Initialisation:
1. $J_{opt} = \infty$, $N_{opt} = 0$, $\theta_{opt} = 0$;
for $j = 1$ **to** $N_{\max}$
  2. Calculate A_j, B_j using Yamanaka-Ankersen (2002) equations;
  3. Solve (2) s.t. $N = j$. If feasible, $J_j = J^*$, else $J_j = \infty$;
  if $J_j < J_{opt}$
    4. $J_{opt} = J_j$, $N_{opt} = j$, $\theta_{opt} = \theta^*$;
  end if
end for.
5. Return u_0 from θ_{opt}.

Spacecraft Application I

Custom QP Solver



- Time varying data
- Varying problem size
- $\implies$ Can't use same design as before
- Still use MINRES
- Mixed software/hardware

Spacecraft Application I

Custom QP Solver System Design

Software on MicroBlaze

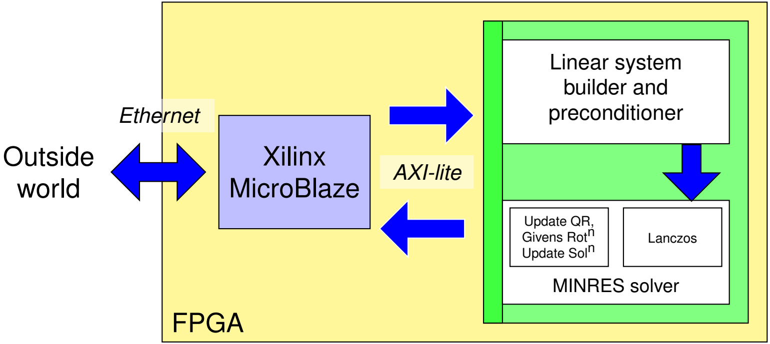
- Iteration counting and control logic
- Prediction model computation
- Some mathematical operations
- Network communications
- Written in C using Xilinx EDK

Custom Circuit

- Communicate with MicroBlaze over AXI
- KKT linearisation and block elimination (RAM latency) and majority of MINRES method
  - Xilinx System Generator for DSP (Graphical), Floating point
- Lanczos algorithm (heavy computation)
  - Simulink HDL Coder (Graphical), Fixed point

Spacecraft Application I

Custom QP Solver Hardware Architecture



Spacecraft Application I

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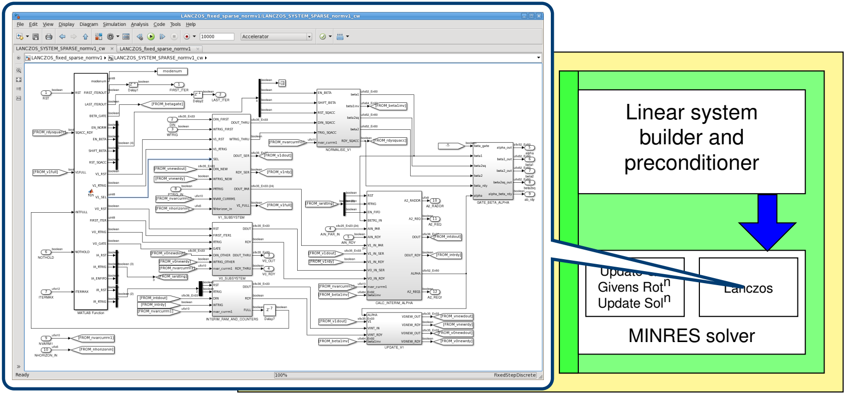

Spacecraft Application I

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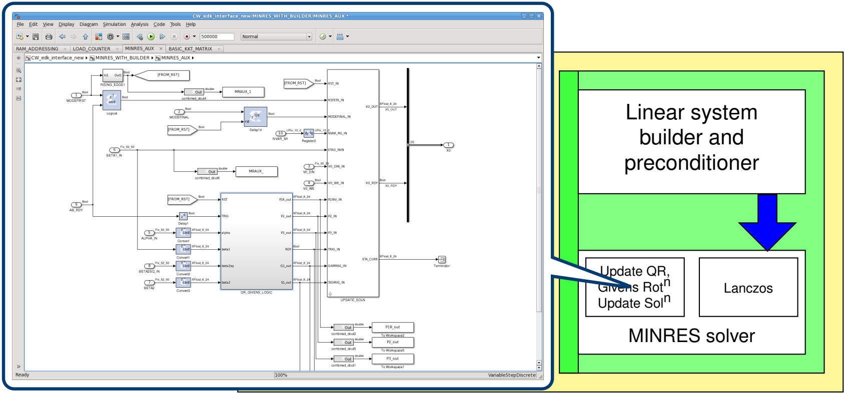

Spacecraft Application I

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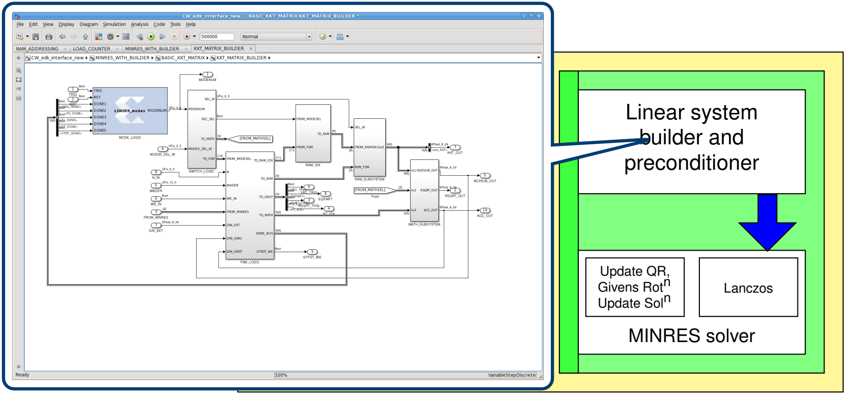
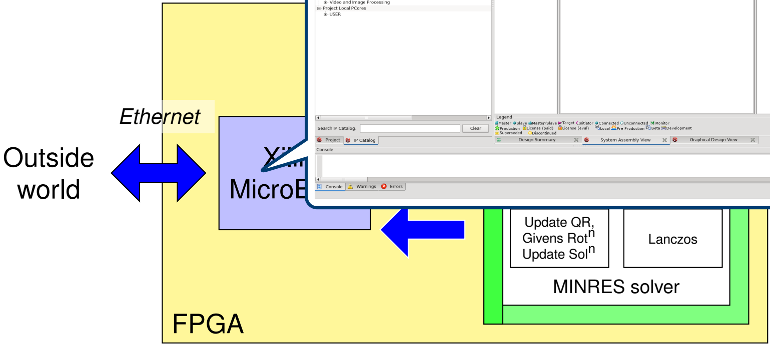
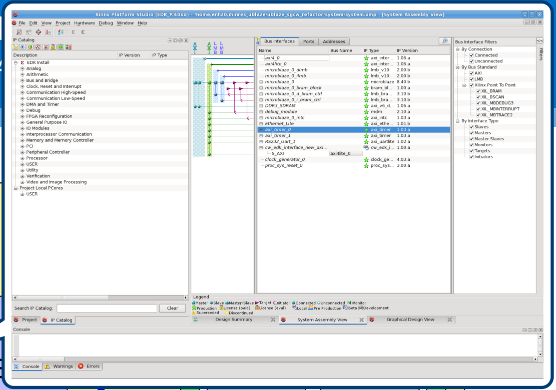


Spacecraft Application I

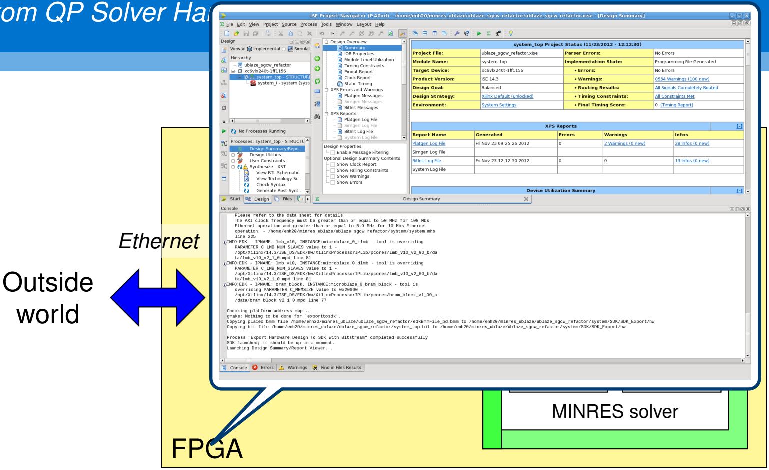
Custom QP Solver Hardware A





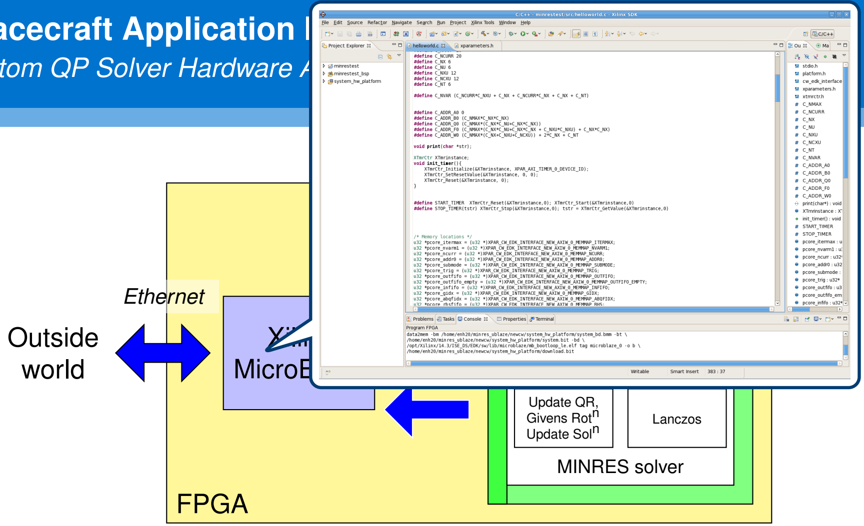
Spacecraft Application I

Custom QP Solver Ha



Spacecraft Application I

Custom QP Solver Hardware A



Spacecraft Application I

Comments on using "high level" graphical tools

Advantages

- Visual documentation
- Simplified communication, maintenance and re-use
- MATLAB/SIMULINK familiar to control engineers
- Rapid prototyping and testing of fixed-point arithmetic
- Still a fair amount of control at the register level

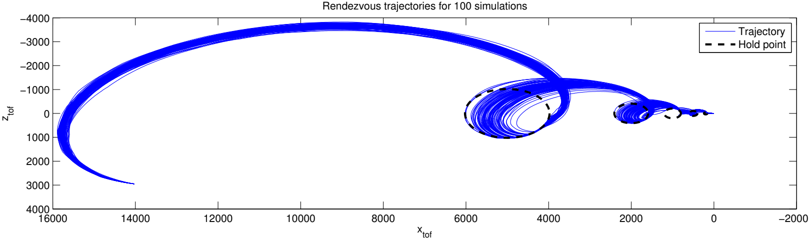
Disadvantages

- Dependent on a longer tool-chain
- Still thinking at a register level
- Possible loss of flexibility *(but can still include custom HDL or IP cores, or combine simpler operations)*
- Division/square root sometimes needs hand-implementation in fixed point to avoid too many layers of logic

Spacecraft Application I

Closed-loop trajectories

FPGA-in-the-loop simulation



- Similar HIL setup as for B747
  - MPC on FPGA, Nonlinear Plant in Simulink
- Plot shows 100 simulations with scattered parameters

Spacecraft Application I

Computation times

Computation time breakdown

		Time (ms)		Alg.	Resource used		
Description	Runs	Once	Total	Step	Microblaze	PCORE	AXI
Compute/transfer LTV model	1	5.01	5.01	–	x		x
Initialise PDIP	1	0.04	0.04	1	x		
Calc. μ_k and w_k	20	0.14	2.82	3–4	x		
Calc. residual	20	0.15	3.05	5	x		
Transfer to PCORE	20	0.11	2.15	5			x
Linear system & MINRES	**20**	**1.00**	**19.94**	6–9		**x**	
Transfer $\Delta\theta_k$	20	0.12	2.36	9			x
Wait for next result	20	0.00	0.02	10	x	x	
Transfer $G(\theta_k + \Delta\theta_k) - g$	20	0.08	1.50	10			x
Sanity check for NaN	20	0.17	3.30	–	x		
Calc. Δs_k, Δz_k	20	0.10	1.92	11–12	x		
Line search	20	0.27	5.44	13	x		
Update solution	20	0.16	3.18	14	x		
Check iter. infeas.	20	0.01	0.19		x		
Check final infeas.	1	0.01	0.01		x		
Total			50.94				

Compare with estimate for full-hardware solver

	Time (ms)			
	$c = 0$	$c = 20$	$c = 30$	$c = 40$
$f_c = 100$ MHz	55.43	62.87	66.59	70.31
$f_c = 200$ MHz	27.71	31.43	33.29	35.15

- Hybrid software/hardware design gives more flexiblity in exchange for minor increase in total solution time

Spacecraft Application I

References

E. N. Hartley and J. M. Maciejowski.
Predictive control for spacecraft rendezvous in an elliptical orbit using an FPGA.
In *Proceedings of the European Control Conference*, pages 1359–1364, Zurich, Switzerland, July 17–19 2013a

E. N. Hartley and J. M. Maciejowski.
Field programmable gate array based predictive control system for spacecraft rendezvous in elliptical orbits.
Optimal Control Applications and Methods, (Article in press), 2014.
doi: 10.1002/oca.2117

- Second paper also considers a terminal phase up to capture by re-purposing the same solver.
- Remaining slides discuss terminal phase but using an alternative approach.

Reference trajectory

- Relative local reference frame centred on target
- At 200 m, accelerate to 0.2 ms^{-1}
- Slow down to 0.1 ms^{-1} at 100 m
- Open-loop drift from 3 m point.

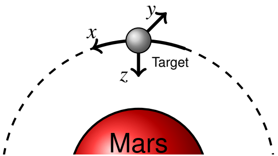


Figure: Relative reference frame

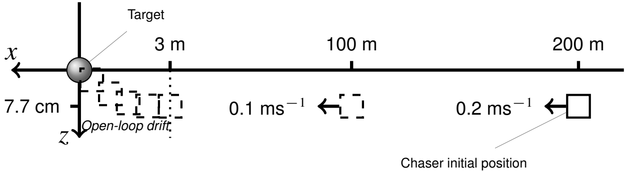


Figure: Nominal reference trajectory (not to scale)

Spacecraft Application II

Problem Description

Constraints and oddities

- Input constraints (8 N)
- Minimum impulse bit

Uncertainty

- Sensor noise
- Thrust uncertainty
- Model parameter uncertainty

Timing

- Sampling time $T_s = 1$ s

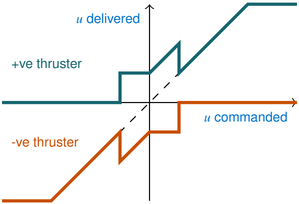


Figure: Differential thrust to counteract minimum impulse bit

Spacecraft Application II

Forming the optimal control problem

- States $x \in \mathbb{R}^5$ (out-of-plane, and nadir relative position), and in-track, out-of-plane and nadir relative velocities
- Inputs $u \in \mathbb{R}^6$ (positive and negative thrusters in three dimensions)
- Prediction matrices (A, B) from Hill-Clohessy-Wiltshire Equations

$$\min_{x_i, u_i} (x_N - r)^T P (x_N - r) + \sum_{i=0}^{N-1} \left((x_i - r)^T Q (x_i - r) + u_i^T R u_i + \|R_\lambda u_i\|_1 \right)$$

Subject to:

$$\begin{aligned} x_0 &= x(k) & \\ x_{i+1} &= Ax_i + Bu_i & i \in \{0, \dots, N-1\} \\ u_i &\geq 0 & i \in \{0, \dots, N-1\} \\ u_i &\leq u_{\max} & i \in \{0, \dots, N-1\}. \end{aligned}$$

- $Q \geq 0$, $R > 0$, $R_\lambda \geq 0$ and diagonal, $P \geq 0$. $N = 20 \implies 120$ decision variables.
- We can write this as a convex, bound constrained Quadratic Program (QP)

Spacecraft Application II

Forming the optimal control problem

- States $x \in \mathbb{R}^5$ (out-of-plane, and nadir relative position), and in-track, out-of-plane and nadir relative velocities
- Inputs $u \in \mathbb{R}^6$ (positive and negative thrusters in three dimensions)
- Prediction matrices (A, B) from Hill-Clohessy-Wiltshire Equations

$$\min_{x_i, u_i} (x_N - r)^T P(x_N - r) + \sum_{i=0}^{N-1} \Big((x_i - r)^T Q(x_i - r) + u_i^T R u_i + \|R_\lambda u_i\|_1 \Big)$$

LASSO term

Subject to:

$$\begin{aligned} x_0 &= x(k) & \\ x_{i+1} &= Ax_i + Bu_i & i \in \{0, \ldots, N-1\} \\ u_i &\geq 0 & i \in \{0, \ldots, N-1\} \\ u_i &\leq u_{\max} & i \in \{0, \ldots, N-1\}. \end{aligned}$$

- $Q \geq 0$, $R > 0$, $R_\lambda \geq 0$ and diagonal, $P \geq 0$. $N = 20 \implies 120$ decision variables.
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Spacecraft Application II

Optimal control problem as a constrained QP

Quadratic Program

$$\min_{\theta} \frac{1}{2}\theta^T H \theta + f^T \theta$$

s.t.

$$\theta_{\min} \leq \theta \leq \theta_{\max}$$

The Good

- Convex
- Efficient solution algorithms (e.g. FGM)

The Not-So-Good

- More complex than simple feedback
- No analytical solution: iterative methods
- At odds with computational constraints

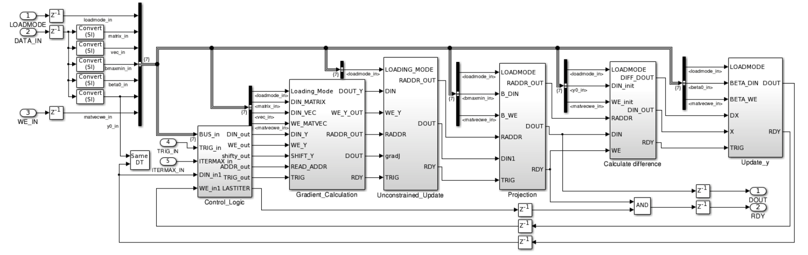
(Power consumption, radiation hardening)

Design Objective

- Solve the QP **fast**, but keep **clock frequency low**

Spacecraft Application II

FPGA Design at a subsystem level using Simulink and HDL Coder



- Fixed point implementation
- Must consider timing. Simulink Delay block $\Longrightarrow$ register in FPGA
- Control logic, counters etc. using MATLAB function blocks
- Includes interface for "on-line" loading of matrices

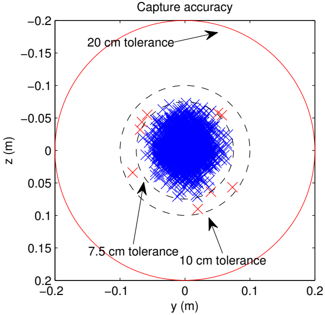
Spacecraft Application II

Synthesis and integration

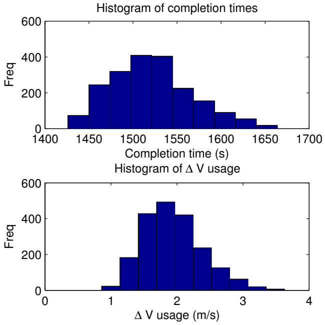
- Use HDL Coder to generate VHDL
- Interface custom VHDL circuit to Xilinx MicroBlaze Soft Core as a memory mapped peripheral
  - Import HDL-Coder generated VHDL into Xilinx SysGen for DSP as "black box", and implement shared memory interface
- Synthesise, place-and-route etc.
- Implement server application (in C) on MicroBlaze, to transfer data from payload of UDP packet to custom circuit and return result
- Test closed loop system (in simulation) connecting PC and Controller via Ethernet

Spacecraft Application II

Integration testing — Monte-Carlo Simulation



(a) $z - y$ position at capture



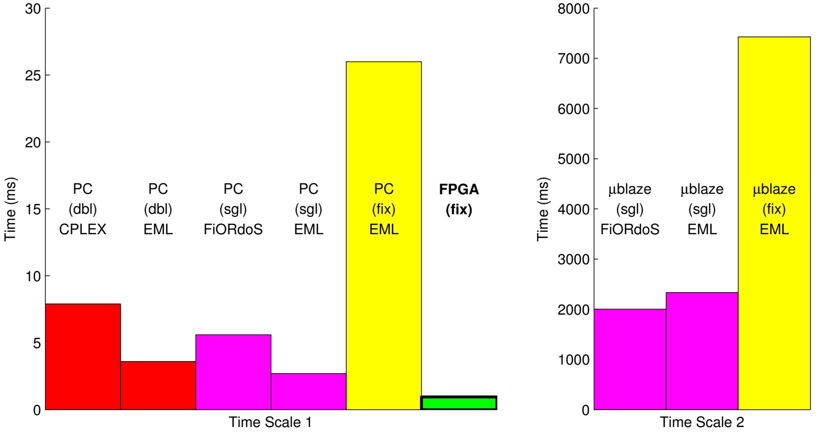
(b) Histograms

Figure: FPGA-in-the-loop Control performance over 2000 random simulations with parameter tuning for 7.5 cm accuracy

- 99.55% within 7.5 cm, 100% within 10 cm

Spacecraft Application II

Comparison of timing



E. N. Hartley, M. Gallieri, and J. M. Maciejowski.
Terminal spacecraft rendezvous and capture using LASSO MPC.
International Journal of Control, 86(11):2104–2113, 2013.
doi: 10.1080/00207179.2013.789608

E. N. Hartley and J. M. Maciejowski.
Graphical FPGA design for a predictive controller with application to spacecraft rendezvous.
In *Proceedings of the Conference on Decision and Control*, pages 1971–1976, Florence, Italy, December 10–13 2013b.
doi: 10.1109/CDC.2013.6760170

MPC for Aircraft

- Interior point method
- Hardware-based MPC solver solution
- MINRES accelerated in hardware

Lessons Learnt

- Possible to implement MPC for quite large problems on FPGA
- Model scaling important for good algorithm performance
- (Having to redo place and route to accommodate model changes is a pain!)

Conclusions (II)

Summary

MPC for Elliptical Spacecraft Rendezvous

- Time-varying prediction model, Variable horizon
- Non-quadratic cost function (but LP = QP with zero Hessian)
- Interior point method
- MINRES accelerated in hardware. Software supervisory logic.
- Hardware design using graphical methods

Lessons Learnt

- Can implement quite complex custom circuits on FPGA for MPC using high level tools based on those familiar to control engineers
- Have to re-implement some basic things like division and square root in fixed point to get decent speed (might have improved?)
- Mixed hardware/software approach flexible with varying problem data — trade off with control latency

Conclusions (II)

Summary

MPC for Terminal Spacecraft Rendezvous

- LTI model, fixed horizon tracking/regulation problem
- LASSO cost function
- First order method
- FPGA circuit design using graphical methods

Lessons Learnt

- First order method quite simple to implement using HDL Coder!
- Fullly fixed point implementation adequately accurate for the application

Thank you for listening!