Trade, firm selection, and innovation: the competition channel*

Giammario Impullitti[†] and Omar Licandro[‡]

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Abstract

The availability of rich firm-level data set has recently led researchers to uncover an interesting set of empirical findings on the effects of trade liberalization. Trade openness forces the least productive firms to exit the market and induces surviving firms to increase their innovation efforts. Together with the selection and the innovation effect, trade liberalization seems to have positive effects on the degree of product market competition. This paper presents a theoretical model aimed at providing a coherent interpretation of these empirical findings. We introduce firm heterogeneity into an innovation-driven growth model. Incumbent firms operating in an oligopolistic environment perform cost-reducing innovation in order to increase their future productivity. In equilibrium more productive firms show higher investment in innovation. The oligopolistic structure implies that markups are endogenously determined by the number of firms competing in same product line. Trade liberalization leads to a higher number of firms and lower markups. This pro-competitive effect of trade forces less efficient firms out of the market and reallocates resources towards more efficient, more innovative, firms; thereby raising aggregate innovation and, consequently, aggregate productivity growth. This dynamic selection effect of trade is decreasing in the level of product market competition and, as a consequence, trade liberalization has negligible effects on innovation in highly competitive economies.

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*Licandro acknowledges the financial support of the Spanish Ministry of Education (SEJ2007-65552).
† *IMT Lucca Institute for Advanced Studies.* Email: g.impullitti@imtlucca.it.
‡ *European University Institute.* Email: omar.licandro@eui.eu.

1 Introduction

An interesting set of empirical regularities has recently emerged from a large numbers of studies using firm-level data. First, empirical evidence has established that large and persistent productivity differences exist among firms within the same industry (e.g. Bartelsman and Doms, 2000). The availability of micro data has also allowed researchers to assess the importance of firm heterogeneity in understanding international trade and its effect on productivity. A number of papers have shown that trade liberalization induces the least productive firms to exit the market and the most productive non-exporter firms to become exporters; this *selection effect* increases the aggregate productivity level (see e.g. Pavcnik, 2002, Topalova, 2004, and Tybout, 2003 for a survey).

A second line of research has focused on the role of firm heterogeneity in shaping the effects of trade liberalization on *innovation* activities affecting the growth rate of productivity. Bustos (2008) shows that a regional trade agreement, MERCOSUR, has selected highly productive firms into exporting and affected positively a broad set of measures of innovation (computers and software, technology transfers, R&D, and patents).[1] Bloom, Draca, and Van Reenen (2008) study the effect of Chinese import penetration on innovation in European countries. They find evidence of both the selection and the innovation effect of trade: on the one hand Chinese competition decreases employment and chances of survival of firms, and this effect is stronger for low-tech than for high-tech firms. On the other hand, surviving establishments tend to innovate more (patenting) and upgrade their technology (IT intensity). Leeiva and Trefler (2008) find that tariff cuts mandated by the Canada-US Free Trade Agreement increase productivity heavily for lower productivity plants, while productivity gains for high productivity plants are negligible. They also show that plants experiencing higher productivity gains are those investing more strongly in innovation and technology upgrading.[2]

A third piece of evidence shows that trade liberalization has *pro-competitive* effects that can potentially lead to more selection and more innovation. Bugamelli, Fabiani, and Sette

[1] Focusing on innovation has the advantage of identifying one specific channel through which improvements in productivity take place. Other studies have instead estimated productivity as a residual in the production function, with the consequence that together with technological differences, residuals captures also other differences such as market power, factor market distortion, and change in the product mix. (see i.e. Foster, Haltowanger, and Syverson, 2008, Hsieh and Klenow, 2008, and Bernard, Redding, and Schott, 2008).

[2] Several paper have investigated the related but slighlty different question of whether the exporter status implies a higher investment in innovation or technology upgrading: this has been called the *learning by exporting* mechanism. The evidence is mixed: early papers, such as Clerides, Lach, and Tybout (1998) and Bernard and Jensen (1999) do not find any evidence in favor of this mechanism. Recent studies have instead found evidence that firm improve their productitivy subsequent to entry (e.g. Delgado, Farinas, and Ruano, 2002, De Loecker, 2006, Van Biesebroek, 2005, see Lopez, 2005, for a survey). The basic difference between these studies and those discussed in the main text is that the former focus on productivity and the latter on innovation. Once exception is Criuscolo, Haskel, and Slaugther (2008) which finds that expoters and multinational firms have higher productivity because they both innovate more and learn from foreign technologies. The other difference is that Bustos (2008), Bloom et al (2008), and Llleiva and Trefler (2008) focus on trade liberalization and not on export status.

(2008) using Italian firm-level manufacturing data find that import competition from China has reduced prices and markups in the period 1990-2004. Griffith, Harrison, and Simpson (2008) have studied the effects of trade integration reforms carried out under the EU Single Market Programme (SMP) and found that these reforms have increased product market competition (measured as average markups) and stimulated innovation (R&D expenditures). Chen, Imbs and Scott (2008) using micro data on EU manufacturing for the period 1989-99, estimate the Ottaviano and Melitz (2008) model and show that trade openness reduces average prices and markups, while raising productivity through firm selection.

This paper presents a theoretical model aimed at providing a coherent interpretation of these empirical findings. We introduce a dynamic industry model with heterogeneous firms into a model of growth with innovation by incumbents. There are two goods in the economy, an homogeneous good produced under constant returns, and a continuum of differentiated goods produced with a variable and a fixed quantity of the homogeneous good. Each variety of the differentiated good is produced by a given number of firms with the same technology, while labor productivity differ across varieties. Thus, as in Hopenhayn (1992) and Melitz (2003) firms are heterogeneous in their productivity. In addition to this now standard environment the model features a dynamic innovation activity performed in-house by firms and aimed at increasing productivity. The market structure for differentiated goods is oligopolistic, thus both the optimal quantity produced and level of innovation result from strategic interaction among heterogenous firms. The oligopolistic market structure and the innovation by incumbents feature are borrowed from static trade models with endogenous market structure (e.g. Neary, 2003 and 2009) and from multi-country growth models with representative firms such as Peretto (2003) and Licandro and Navas (2008).

The open economy features two symmetric countries engaging in costly trade (iceberg type). In order to simplify the analysis, we assume that there are no entry costs into the export market, implying that all operating firms export. The fixed production costs and the heterogeneous firms structure determine the cutoff productivity level below which firms cannot profitably produce. Trade liberalization (reduction in trade costs) leads to an increase in product market competition because the number of firms producing each variety doubles. This yields a reduction in the markup and a decrease in the inefficiency of oligopolistic markets, ultimately leading to an expansion of the quantity produced by each firm. A decline in the markup raises the productivity cutoff and forces the less productive firms out of the market. The basic result of the paper is that firm selection affects aggregate innovation according to the following mechanism: an increase in the productivity cutoff implies that resources are reallocated from exiting firms to higher productivity surviving firms, which are also those innovating at a higher pace. Thus, trade-induced firm selection increases not only aggregate productivity (as in Melitz, 2003) but also aggregate innovation, which contributes to increase the growth rate of productivity. We call this new mechanism the *dynamic selection* effect. Secondly, the pro-competitive effects of trade has also a *direct effect* on innovation produced by the increase in quantity: since innovation is cost reducing, the marginal benefit from a reduction in costs is increasing with the quantity produced. Both effects are decreasing in the level of product market competition of the economy before trade liberalization: in highly competitive economies (large number of firms) trade liberalization has only negligible effects on innovation, and there exist a threshold level of competition above which trade has no effects on innovation.

This paper is related to the emerging literature studying the joint selection and innovation effect of trade openness. A first line of research introduces a one-step technological upgrading choice into an heterogeneous firm framework. Examples are Yeaple (2005), Costantini and Melitz (2007), Bustos (2007), Navas and Sala (2007), Vannoorenberge (2008). In all these papers, with exception of Costantini and Melitz, the model economy is static, and the dynamic effects of trade on innovation cannot be analyzed. Our paper is more closely related to a second line of research that introduces innovation as a continuous process in dynamic models of trade and productivity growth. Baldwin and Robert-Nicoud (2008) and Gustaffson and Segerstrom (2008) explore the effects of trade liberalization on innovation and growth in models of expanding variety (Romer, 1990) with heterogeneous firms. They show that the effect of trade-induced firm selection on innovation and growth depends on the form of (international) knowledge spillovers characterizing the innovation technology. Atkeson and Burnstein (2007) set up a model of process and product innovation with firm heterogeneity and show that trade has positive effects on process innovation that can be offset by negative effects on product innovation.[3]

Although differing on the type of innovation or on the specific form of innovation technology they analyze, all these papers adopt a monopolistically competitive market structure.[4] The key distinguishing feature of our model is that we study the interactions between trade, firm heterogeneity and innovation in an oligopolistic environment. In this framework the market structure is endogenous and responds to changes in trade costs, thereby representing the ideal environment to analyze the effects of trade on product market competition (the third stylized fact discussed above). Melitz and Ottaviano (2008) show that with a special assumption on preferences it is possible to obtain endogenous markups in the monopolistic competitive framework. In line with our result, they find that trade liberalization produces a pro-competitive effect (lower markups). It worth noticing that the presence of endogenous markups allows the selection effect to work through a channel different from that highlighted in Melitz (2003). In

[3] Klette and Kortum (2004) and Mortensen and Lentz (2008) introduce a dynamic industry model with heterogeneous firms into a quality ladder growth model (Grossman and Helpman, 1991). They limit the analysis to the interaction between firm heterogeneity and creative destruction in closed economy, without exploring the effects of trade. Haruyama and Zhao (2008) explore the interaction between trade liberalization, selection and creative destruction.

[4] One exeption is Van Long, Raff, and Stahler (2008) that features an oligopolistic market structure, but innovation is not a continuos process and the model is static.

that paper, trade liberalization produces an increase in labor demand that bids up wages and forces low productivity firms to exit. In our paper, as in Melitz and Ottaviano (2008), the selection effect is produced by the reduction in markups brought about by trade liberalization. While there is evidence, as discussed above, that trade liberalization has increased product market competition, the trade-induced increase in average wages triggering firm selection in Melitz (2003) seems to be counterfactual.[5] Our model differs from that of Melitz and Ottaviano not only for the different source of endogenous markups but also because in their model there is no innovation activity aimed at improving productivity, therefore they cannot study the implications of firm heterogeneity and endogenous markups for innovation. Bernard, Jensen, Eaton, and Kortum (2003), set up a Ricardian model with Bertrand competition among firms and obtain markups responding endogenously to trade liberalization. We complement their analysis by introducing innovation and deriving endogenous markups from Cournot competition.

Summing up, to our knowledge the present paper is the first to provide a framework to interpret all three stylized facts discussed above. The basic structure of the model is such that trade affects both firm selection and innovation through the *competition channel*. The selection effect of trade operating through endogenous markups resulting from oligopolistic competition among firms is a novel contribution. Secondly, while the direct competition effect of trade on innovation is not new in the literature (see Licandro and Navas, 2007), the interaction between firm selection and innovation represents an original contribution of this paper. Finally, although our stylized economy features a rich and complex structure (oligopoly, firm heterogeneity, growth), the model is highly tractable and all results are obtained analytically, with no special assumption on either preferences or firms productivity distribution.

2 The model

2.1 Economic environment

The economy is populated by a continuum of identical consumers of unit measure. Time is continuous and denoted by t, with initial time $t = 0$. Initial conditions are such that the economy is at a stationary equilibrium, if it exists, from the initial time.

Preferences of the representative consumer are

$$\int_0^\infty (\ln X_t + \beta \ln Y_t)\ \mathrm{e}^{-\rho t}\ \mathrm{d}t,$$

with discount factor $\rho > 0$. There are two types of goods, an homogeneous good, taken as the numeraire, and a differentiated good. Consumers are endowed with a unit flow of the homogeneous good. A fraction Y of it is consumed, entering utility with weight $\beta > 0$.

[5]For instance March CPS data show that both median and average US wages have stagnated in the last three decades, a period of progressive trade liberalization (see Acemoglu, 2002)

The differentiated good X is produced with a continuum of varieties of mass $M \in [0,1]$ according to

$$X_t = \left(\int_0^{M_t} x_{jt}^{\alpha} \, \mathrm{d}j \right)^{\frac{1}{\alpha}}, \tag{1}$$

where x_{jt} represents variety j, and $\sigma = \frac{1}{1-\alpha}$ is the elasticity of substitution across varieties with $\alpha \in (0,1)$. Each variety in X is produced by n identical firms by transforming the homogeneous good into this particular variety. Firms face the same fixed production cost $\lambda > 0$ but may have different productivities z. A firm with productivity z has the following production technology (we omit index j)

$$q_t = (z_t + b\hat{z}_t)(y_t - \lambda), \tag{2}$$

where y represent inputs and q production. Variable $\hat{z}$ is the average productivity of the remaining $(n-1)$ firms producing the same variety and parameter $b > 0$, represents productivity spillovers from other firms producing the same variety.

Innovation activities are undertaken by incumbents according to the following technology

$$\dot{z}_t = A \ \hat{z}_t h_t, \tag{3}$$

where h represents units of the homogeneous good allocated to innovation and innovation efficiency is denoted by $A > 0$. R&D activities are also assumed to benefit from spillovers coming from direct competitors. Let assume, for simplicity, that all firms producing the same good have the same initial productivity $z_0 > 0$.

Irrespective of their productivity z, varieties exit the market at rate $\delta > 0$. On top of that, exit may result from productivity z being too low. This is the selection effect described in section 2.4 below. Exiting varieties are replaced by new varieties in order to the mass of varieties remains constant. The productivity distribution of entrants is described in section 2.5.

2.2 Households

The representative household maximizes utility subject to its instantaneous budget constraint. The solution to this problem is straightforward and leads to the following first order conditions

$$Y_t = \beta E_t, \tag{4}$$

$$\frac{\dot{E}_t}{E_t} = r_t - \rho, \tag{5}$$

$$p_{jt} = \frac{E}{X_t^{\alpha}} \, x_{jt}^{\alpha-1}, \tag{6}$$

where r is the interest rate and p_{jt} is the price of good j. Total expenditure on the composite good X is

$$E_t = \int_0^{M_t} p_{jt} x_{jt} \, \mathrm{d}j.$$

Because of log preferences, total spending in the homogeneous good is β times total spending in the differentiated good. Equation (5) is the standard Euler equation, and (6) is the inverse demand function for variety j, $j \in [0, 1]$.

2.3 Production and Innovation

Firms producing the same good behave non-cooperatively and maximize the present value of their net cash flow.

$$V_{is} = \int_s^{\infty} \pi_{it} R_t dt,$$

where R_t is the discount factor and $\pi_{it} = (p_{it} - (z_{it} + b\hat{z}_t)^{-1})q_{it} - h_{it} - \lambda$ is the profit. We solve this differential game focusing on Nash Equilibrium in open loop strategies. Let $a_i = (q_{it}, h_{it})$ for $t \geq s$ be a strategy for firm i. These strategies are time paths for quantity and R&D. In the open loop equilibrium we construct firms commit to time paths strategies for quantities and R&D, which induce time paths for productivity. At time s a vector of strategies $(a_1, ..., a_i, ..., a_n)$ is an equilibrium if

$$V_{is}(a_1, ..., a_i, ..., a_n) \geq V_i(a_1, ..., a_i', ..., a_n) \geq 0$$

where $(a_1, ..., a_i', ..., a_n)$ is the vector in which only firm i deviates from the equilibrium path of quantity and R&D. The first inequality states that firm i maximize its present value of its net cash flow, and the second condition requires this to be positive.[6]

The characterization of the open loop Nash equilibrium proceeds as follows: a firm producing a particular good solves at any time s the problem

$$V_s = \max_{(q_t, h_t)_{t=s}^{\infty}} \int_s^{\infty} \left[(p_t - (z_t + b\hat{z}_t)^{-1})q_t - h_t - \lambda\right] e^{-(\rho+\delta)(t-s)} \, dt, \qquad \text{st.} \tag{7}$$

[6] The open loop equilibrium concept does not generally have the property of subgame perfection, as firms choose their optimal time-paths strategies at the initial time and stick to them forever. In closed loop strategies, instead, firms do not pre-commit to any path and their strategies at any time depend on the whole past history. The Nash equilibrium in this case is strongly time-consistent and therefore sub-game perfect. The literature on differential games has uncovered classes of games in which the open loop equilibrium degenerates into a closed loop and therefore is subgame perfect (e.g. Reingaum, 1982 and Fershtman,1987). Cellini and Lambertini (2004) set up a dynamic oligopoly game with cost-reducing R&D and R&D spillovers similar to the one we solve below. They show that the open loop solution collapse into the closed loop when in the first order condition for a firm's control variable other firms' state variables are not present. In our model this condition is satisfied only setting the production externality $b = 0$. This would not affect any of our results but would require stronger parameter restrictions - as we will see later.

$$
\begin{aligned}
p_t &= \frac{E_t}{X_t^{\alpha}}\, x_t^{\alpha-1} \\
x_t &= \hat{x}_t + q_t \\
\dot{z}_t &= A\ \hat{z}_t\ h_t \\
z_s &> 0,
\end{aligned}
$$

where $\delta > 0$ is the exogenous exit rate. In a Cournot game a firm takes as given the path of its competitors' production $\hat{x}_t$, the path of its competitors' average productivity $\hat{z}_t$, as well as the path of the aggregates E_t and X_t. The first order conditions for the problem above are

$$(z_t + b\hat{z}_t)^{-1} = \theta \underbrace{\frac{E}{X_t^{\alpha}}\, x_t^{\alpha-1}}_{p_t}, \tag{8}$$

$$1 = v_t A \hat{z}_t, \tag{9}$$

$$\frac{(z_t + b\hat{z}_t)^{-2}}{v_t} q_t = \frac{-\dot{v}_t}{v_t} + \rho + \delta, \tag{10}$$

where $\theta \equiv (n - 1 + \alpha)/n$ is the inverse of the markup rate. It is easy to see from (8) that firms charge a constant markup $1/\theta$ over marginal costs $(z_t + b\hat{z}_t)^{-1}$. As it is well-known, in a Cournot-type equilibrium the markup depends not only on demand elasticity but on the number of competitors.

Note that firms producing the same good operate the same technology and face the same initial conditions, implying that at equilibrium $\hat{z}_t = z_t$ and $x_t = nq_t$. After some simple algebra, it can be shown that the equilibrium growth rate of productivity follows

$$\frac{\dot{z}_t}{z_t} = \frac{A}{1+b} \frac{q_t}{z_t(1+b)} - \rho - \delta, \tag{11}$$

$$\frac{q_t}{z_t(1+b)} = \theta e_t \left(\frac{z_t}{\overline{z}_t}\right)^{\frac{\alpha}{1-\alpha}}, \tag{12}$$

where $e_t = E_t/nM_t$ is expenditure per firm and average productivity is defined as

$$\overline{z}_t = \frac{1}{M_t} \left(\int_0^{M_t} z_{jt}^{\frac{\alpha}{1-\alpha}}\, \mathrm{d}j \right)^{\frac{1-\alpha}{\alpha}}.$$

To obtain the growth rate of productivity in (11), differentiate (9) and substitute the resulting $\frac{\dot{v}}{v}$ in (10), then substitute $v_t z_t$ from (9). Equation (12) derives from (8) and (1) after substituting $x_t = nq_t$.

When innovation is cost reducing and undertaken by incumbents, profitability of R&D depends on production, as shown by the right hand side in (10). The benefit of reducing production costs is larger, the larger is production. Since more efficient firms produce more, they also have more incentives to do R&D, meaning that firm's R&D activity depends positively on firm's state of technology. This is reflected in equation (11), where the growth rate of productivity depends positively on output, which depends on the normalized productivity level.

The growth rate of the average productivity $\overline{z}_t$, which we denote by g, is

$$g_t = \frac{A\theta e_t}{1+b} - \rho - \delta. \tag{13}$$

The term $A\theta e_t/(1+b)$ is the marginal return to R&D investment for firms with productivity $z_t = \overline{z}_t$. As time passes, firms with productivity initially smaller than the mean will grow at a smaller and smaller rate, but firms with productivity initially larger than the mean will keep growing at a growing rate.

2.4 Exit

Using results from firms optimization problem derived above, it can be easily shown that firm z's cash flow is

$$\pi(z_t) = \left(\frac{1}{\theta} - 1\right)\frac{q_t}{z_t\,(1+b)} - \underbrace{\left(\frac{1}{1+b}\frac{q_t}{z_t\,(1+b)} - \frac{\rho+\delta}{A}\right)}_{h} - \lambda. \tag{14}$$

From (12), quantities q depend on the distance from average productivity $\tilde{z}$, defined as $\tilde{z}_t = \left(\frac{z_t}{\overline{z}_t}\right)^{\frac{\alpha}{1-\alpha}}$. Consequently, we can denote $\pi = \pi(\tilde{z})$. An interesting property of a model with innovation by incumbents is that an increase in competition, as measured by θ, by reducing prices and increasing quantities, may have a positive effect on innovation. This positive effect is related to the fact that increased production makes a cost-reducing innovation more profitable. However, this positive effect of competition on innovation is related to a decreasing cash flow, not only because margins go down but because firms invest more in R&D. Let rewrite the cash flow in (14) as

$$\pi(z_t) = \left(\frac{1}{\theta} - \frac{2+b}{1+b}\right)\frac{q_t}{z_t\,(1+b)} + \frac{\rho+\delta}{A} - \lambda. \tag{15}$$

Note that produced quantities, and then $\tilde{z}$, affect π positively if and only if $b > \frac{2\theta-1}{1-\theta}$, which is assumed in the following. When markets are highly competitive, say $\theta < \frac{1}{2}$, any externality $b \geq 0$ supports an equilibrium where the cash flow depends positively on output. However, when θ approaches unity, an arbitrarily high externality is required to profits depend positively on quantities.[7]

From (12) we know that $q_t/z_t\,(1+b) = \theta e_t \tilde{z}_t$. Let us call $\tilde{z}_t^*$ to the cutoff relative productivity below which firms exit the market. The productivity at time t of a marginal firm, i.e. a firm

[7] If we assume $b = 0$, as we mentioned above, the open loop equilibrium we consider will coincided with a closed loop equilibrium, therefore being subgame perfect. But for the profit function to be increasing in productivity $\tilde{z}$ we need to put a strong restriction on markups: $\theta < \frac{1}{2}$, implying a markup larger than 2. All basic results of the paper are not affected by the assumption $b = 0$, but we decided to keep $b > 0$ to avoid restricting the markup to high values.

with relative productivity $\tilde{z}_t = \tilde{z}_t^*$, grows at a lower rate than the mean, implying that it will exit the market at $t+$ dt. For this reason, profits of the marginal firm have to be zero, that is

$$\left(1 - \frac{2+b}{1+b}\theta\right) e_t \tilde{z}_t^* + \frac{\rho+\delta}{A} - \lambda = 0. \tag{16}$$

The marginal firm is making zero profits the time just before exiting.[8]

2.5 Stationary Equilibrium

In order to characterize the stationary equilibrium we need to introduce the entry process of new varieties and market clearing conditions. We assume there is a mass of unit measure of potential sectors, of which $M \in [0,1]$ are operative. New varieties can enter the market at zero cost and draw a productivity from the initial productivity distribution $F(\tilde{z})$. Since firms exit at the rate δ, stationarity requires

$$(1 - M)(1 - F(\tilde{z}^*)) = \delta M. \tag{17}$$

This condition says that the exit flow δM equals the entry flow defined by the number of entrants $1 - M$ times the probability of surviving $1 - F(\tilde{z}^*)$. Consequently, the mass of operative firms M is a decreasing function of the cutoff level $\tilde{z}^*$,

$$M(\tilde{z}^*) = \frac{1 - F(\tilde{z}^*)}{1 + \delta - F(\tilde{z}^*)}. \tag{18}$$

The market clearing condition for the homogeneous good can be written as

$$n \int_0^M (y_j + h_j) \ \mathrm{d}j + Y = n \int_0^M \left(\frac{q_j}{z_j (1+b)} + h_j + \lambda \right) \ \mathrm{d}j + \beta E = 1,$$

In steady state Y and E are constant. The (measure one) total endowment of the homogeneous good is allocated to composite good production and innovation, as well as to homogeneous good consumption. The first equality comes after substitution of y from (2), and Y from (4).

Let $\mu(\tilde{z})$ be the stationary density distribution of firms defined in the $\tilde{z}$ domain.[9] The endogenous exit process related to the cutoff point $\tilde{z}^*$ implies $\mu(\tilde{z}) = 0$ for all $\tilde{z} < \tilde{z}^*$. Rewriting the labor market clearing condition after changing the integration domain from sectors $j \in [0,1]$ to productivity $\tilde{z} \in [\tilde{z}^*, \infty]$, and substituting R&D employment from (3) and (11), and $\frac{q}{z(1+b)}$ from (12) we obtain

$$\int_{\tilde{z}^*}^{\infty} \left(\frac{2+b}{1+b} \ \theta e \tilde{z} - \frac{\delta+\rho}{A} + \lambda \right) \ \mu(\tilde{z}) \ d\tilde{z} + \beta e = \frac{1}{nM}.$$

[8] In fact, firms with initial productivity smaller than the mean face an endogenous finite life, which is reached when productivity becomes equal to the cutoff level. It corresponds to an optimal control problem with horizontal terminal line. The associated terminal condition requires profits been zero at the terminal time. See section 7.4 in Chiang (1992).

[9] Proposition 5 in the appendix provides necessary and sufficient conditions for a stationary distribution to exists.

Since $\int_{\tilde{z}^*}^{\infty} \mu(\tilde{z})\, d\tilde{z} = \int_{\tilde{z}^*}^{\infty} \tilde{z}\mu(\tilde{z})\, d\tilde{z} = 1$, after integrating over all sectors we obtain

$$e = \frac{\frac{1}{nM(\tilde{z}^*)} + \frac{\delta+\rho}{A} - \lambda}{\beta + \frac{2+b}{1+b}\theta}. \tag{MC}$$

The other equilibrium condition is determined by the exit condition (16)

$$e\tilde{z}^* = \frac{\lambda - \frac{\rho+\delta}{A}}{1 - \frac{2+b}{1+b}\theta} \tag{EC}$$

Since M is decreasing in $\tilde{z}^*$, therefore (MC) and (EC) are respectively increasing and decreasing in $(e, \tilde{z}^*)$. The following proposition establishes parameter conditions under which the solution for e and $\tilde{z}^*$ is interior.

Assumption 1 *The following parameter restrictions hold*

$$b > \frac{2\theta - 1}{1 - \theta} \tag{a}$$

$$\frac{1}{n} + \frac{\delta+\rho}{A} > \lambda > \frac{\rho+\delta}{A} \tag{b}$$

$$\beta > \frac{\frac{1}{nM(1)}\left(1 - \frac{2+b}{1+b}\theta\right) - \left(\lambda - \frac{\rho+\delta}{A}\right)}{\lambda - \frac{\rho+\delta}{A}} \tag{c}$$

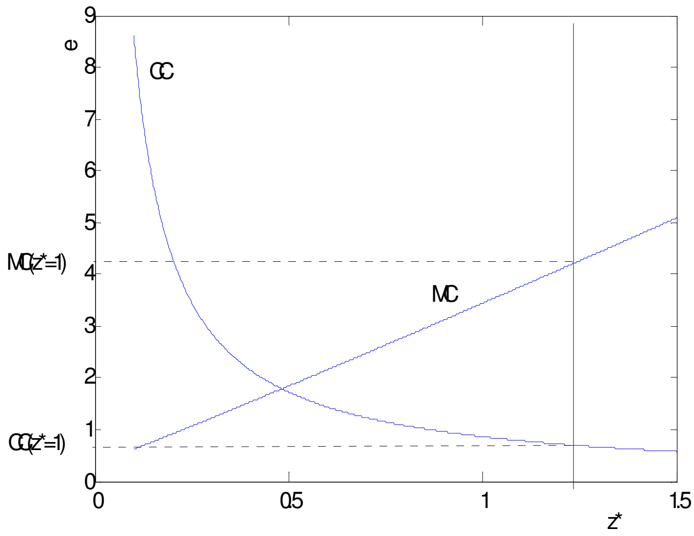
Assumption (a) makes the profit function (15) increasing in $\tilde{z}$. This assumption, together with (b), makes strictly positive the right-hand-side of both (MC) and (EC). As shown below, assumption (c) bounds the equilibrium cutoff $\tilde{z}^*$ below 1, coherently with the definition of $\tilde{z}$.

Proposition 1 *Under Assumption 1, there exists a unique interior solution* $(e, \tilde{z}^*)$ *of (MC)-(EC)*

Proof. From (MC), e is an increasing function of $\tilde{z}^*$ and from (EC), e is an decreasing function of $\tilde{z}^*$. In (EC), e goes to infinity when $\tilde{z}^*$ goes to zero, and under Assumption 1 (c), at $\tilde{z}^* = 1$ e is larger in (MC) than in (EC). Consequently, the locus $(e, \tilde{z}^*)$ in (MC) and (EC) cross once and only once for $\tilde{z}^* \in (0, 1)$. ■

Figure 1 below illustrates the proof of Proposition 1

Figure 1. Steady state equilibrium

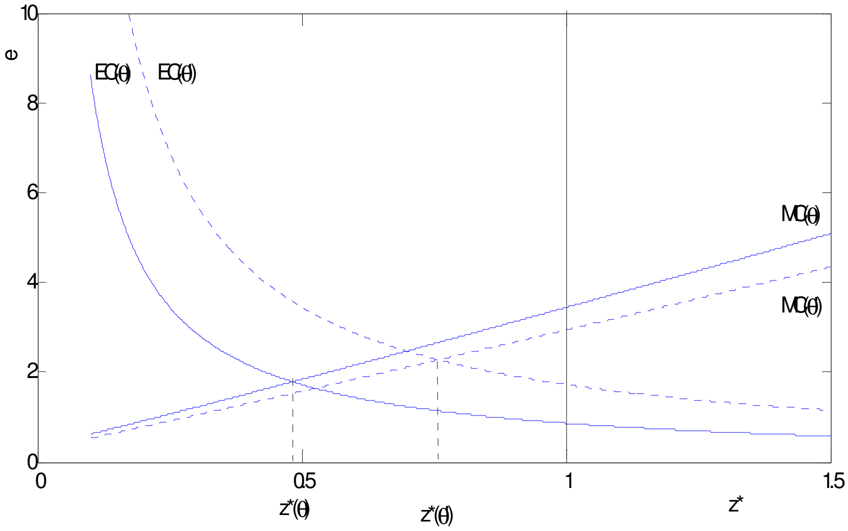


(CC) is the exit condition (EC) and (MC) is the market clearing condition (MC). Assumption (c) is sufficient for $CC(\tilde{z}^* = 1) < MC(\tilde{z}^* = 1)$, thereby (CC) always crosses (MC) for $\tilde{z}^* < 1$.

Proposition 2 *An increase in* θ *raises the productivity cutoff* $(d\tilde{z}^*/d\theta > 0)$*, reduces the number of operative varieties* $(dM(\tilde{z}^*)/d\theta < 0)$*, has an ambiguous effect on the labor resources allocated to the homogeneous sector* $(d(\beta e)/d\theta < 0)$ *and increases the growth rate* $(dg/d\theta > 0)$

Figure 2 shows the effect of an increase in the degree of competition (reduction in the markup $1/\theta$) on the equilibrium values of $\tilde{z}^*$ and e. An increase in θ shifts both the (CC) and the (MC) curves to the right, thereby increasing the equilibrium productivity cutoff $\tilde{z}^*$. Depending on the relative strengths of the shift of the two curves e can increase or decrease, but the average growth rate g always increases. Intuitively, from (13) we know that the effect of a change in θ on g is determined by its effect on θe. Multiplying the market clearing condition (MC) by θ we can obtain θe as a function of θ and $M(\tilde{z}^*)$, and since in equilibrium $M(\tilde{z}^*)$ is decreasing in θ, we can conclude that θe is increasing in θ.

Figure 2. Increasing competition



Two mechanisms contributes to increasing growth, a direct competition effect and a selection effect. Let describe first the *direct competition effect.* In a Cournot equilibrium, an increase in competition reduces markups and allows for an increase in produced quantities. The increase in quantities is feasible since the homogeneous good becomes relatively more expensive (i.e. the relative efficiency of the differentiate sector increases), consumers' demand moves away from it towards the composite good and resources are reallocated from the homogeneous to the composite sector. Since the payoff of cost-reducing innovation is increasing in the quantity produced, the higher static efficiency associated to lower markups brought about by competition affects positively innovation and growth. This mechanism does not depend on firm heterogeneity: it is easy to check that assuming away the dependence of M on $\tilde{z}^*$ by setting $M = 1$, the equilibrium growth rate derived from (MC) and (EC) becomes independent on the cutoff $\tilde{z}^*$, but still increasing in θ. This direct effect of competition on growth can in fact be found in representative firm models of growth with endogenous market structure (see e.g. Peretto, 2003, and Licandro and Navas, 2007).

The *selection effect* is instead specifically related to the heterogeneous firm structure of the model. The trade-induced reduction in the markup raises the productivity threshold above which firms can profitably produce, the cutoff $\tilde{z}^*$, thus forcing the least productive firms to exit the market. Resources are reallocated from exiting firms to the higher productivity surviving firms which, as shown in (11), innovate at a higher pace. Therefore this selection effect leads to higher innovation and growth. Notice that in this model the direct competition effect of trade liberalization on innovation does not hold if we eliminate the homogeneous good, because no reallocation of market shares would be possible. While the selection effect produced by the presence of firm heterogeneity would still hold because reallocation takes place within varieties of the differentiated product.

3 Open economy

Consider a world economy populated by two symmetric countries with the same technologies, preferences, and endowments as described in the previous section. We assume that trade costs are of the iceberg type: $\tau > 1$ units of goods must be shipped abroad for each unit finally consumed. Costs τ can represent transportation costs or trade barriers created by policy. For simplicity we do not assume entry costs in the export market, thus all surviving firms sell both to the domestic and foreign markets.[10]

3.1 Equilibrium characterization

Since the two countries are perfectly symmetric, we can focus on one of them. Let q_t and $\check{q}_t$ be the quantities produced for the domestic and the foreign markets, respectively. The firm solves a problem similar to that in closed economy (see appendix). The first order conditions are:

$$\begin{aligned}
((1+b)z_t)^{-1} &= \left((\alpha-1)\frac{q_t}{x_t}+1\right)p_t \\
\tau\left((1+b)z_t\right)^{-1} &= \left((\alpha-1)\frac{\check{q}_t}{x_t}+1\right)p_t \\
1 &= v_t A z_t, \\
\frac{((1+b)z)^{-2}}{v_t}\left(q_t+\tau\check{q}_t\right) &= \frac{-\dot{v}_t}{v_t}+\rho+\delta.
\end{aligned}$$

Firms face different marginal costs and set different markups for the domestic and foreign markets. In the appendix we show that the first two of the conditions above yield the following equilibrium quantities for each firm

$$\frac{q_t+\tau\check{q}_t}{(1+b)z_t} = A\theta^T e_t \tilde{z}_t \tag{19}$$

where

$$\theta^T = \frac{2n-1+\alpha}{n\left(1+\tau\right)^2\left(1-\alpha\right)}\left[\tau^2\left(1-n-\alpha\right)+n\left(2\tau-1\right)+\left(1-\alpha\right)\right] \tag{20}$$

is the inverse of the average markup in the open economy. Notice that θ^T is a decreasing function of the variable trade cost parameter τ, with θ^T reaching its maximum value $\theta^T_{\max} \equiv \left(2n-1+\alpha\right)/2n$ when $\tau = 1$, the polar case of no iceberg trade costs; and the autarky value $\theta = \left(n-1+\alpha\right)/n$ when $\tau = n/\left(n+\alpha-1\right)$, the alternative polar case where trade costs are prohibitive and economies do not have incentives to trade.

[10] Our main goal is to explain the interaction between trade, selection and innovation, and for this purpose having firms partitioned by their export status is not necessary.

Using the last two first order conditions above and proceeding as in the closed economy, we find that the growth rate of productivity

$$\frac{\dot{z}_t}{z_t} = \frac{A}{(1+b)} \left(\frac{q_t + \tau \breve{q}_t}{(1+b) z_t} \right) - \rho - \delta = \frac{A}{(1+b)} \theta^T e \tilde{z} - \rho - \delta \tag{21}$$

takes the same functional form as in the closed economy. Consequently, opening to trade only affects equilibrium growth rates through changes in the markup.

As in the closed economy case, we focus on the characterization of the steady-state equilibrium. The productivity cutoff is determined solving the following equation

$$\pi(\tilde{z}^*) = \left(p - \frac{1}{\tilde{z}^*(1+b)} \right) q + \left(p - \frac{\tau}{\tilde{z}^*(1+b)} \right) \breve{q} - h - \lambda = 0$$

which, as shown in the appendix, yields

$$e^T \tilde{z}^{*T} = \frac{\lambda - \frac{\rho+\delta}{A}}{1 - \left(\frac{2+b}{1+b}\right) \theta^T}. \tag{EC$^{\mathrm{T}}$}$$

Since firms compensate their losses in local market shares by their new shares in the foreign market, profits are only affected by the change in the markup. Consequently, the exit condition has the same functional form as in (EC) except for the θ^T.

The market clearing condition becomes

$$n \int_0^M \left(\frac{q_j}{z_j (1+b)} + \frac{\tau \breve{q}_j}{z_j (1+b)} + \left(\frac{q_j + \tau \breve{q}_j}{z_j (1+b)} \frac{1}{1+b} - \frac{\rho + \delta}{A} \right) + \lambda \right) \mathrm{d}j + \beta E = 1,$$

which, proceeding as in the closed economy case yields

$$e^T = \frac{\frac{1}{nM(\tilde{z}^{*T})} + \frac{\delta+\rho}{A} - \lambda}{\beta + \frac{2+b}{1+b} \theta^T}. \tag{MC$^{\mathrm{T}}$}$$

which is equal in all aspects to (MC) except for the markup, with θ^T instead of θ. Equations (EC$^{\mathrm{T}}$) and (MC$^{\mathrm{T}}$) yield the equilibrium $(e^T, \tilde{z}^{*T})$ in the open economy. The equilibrium growth is defined by (13) with θ^T and e^T instead of θ and e.

Proposition 3 *Under Assumption 1 and for* $\tau < \bar{\tau} = \frac{n}{n+\alpha-1}$ *there exists a unique interior solution* $\left(e^T, \tilde{z}^{*T}\right)$ *of (MC^T) and (EC^T).*

Proof. At $\bar{\tau} = n/(n + \alpha - 1)$ the markups under trade and autarky are equal, $\theta^T = \theta$, and the prohibitive level of trade costs is reached. Thus, for $\tau \geq \bar{\tau}$ firms do not have incentives to export, and trade does not take place. For $\tau < \bar{\tau}$ the proof of the existence is similar to that in the closed economy, and we omit it for brevity. ∎

3.2 Trade liberalization

Since (MC^T) and (EC^T) are formally equivalent to (MC) and (EC) apart from θ, we can apply Proposition 2 to study the effects of trade liberalization. Trade openness does not affect market shares because the increase in the number of firms in the domestic market is offset by the access to the export market. The economy with costly trade is characterized by a level of product market competition higher than in autarky, $\theta^T > \theta$. A larger number of firms in the domestic market, raises product market competition, thus lowering the markup rate. From the definition of θ and the equilibrium value of θ^T we obtain

$$\theta^T - \theta = \frac{\tau (1-\alpha)^2 - n (\tau - 1)^2 (n + \alpha - 1)}{n (1+\tau)^2 (1-\alpha)}.$$

For $\tau < \bar{\tau}$ the markup under trade is lower, that is $\theta^T - \theta > 0$, and by differentiating the expression above it is easy to see that the distance between θ^T and θ is decreasing in τ. Hence, trade liberalization increases product market competition. When trade is completely free, $\tau = 1$, product market competition reaches its maximum level, $\theta^T_{\max} \equiv (2n - 1 + \alpha)/2n$. Notice that $\theta^T_{\max}$ has the same functional form as the inverse of the markup in autarky but with the number of firms doubled.

Once established that trade reduces markups, from (EC^T) trade liberalization increases the productivity threshold for firms to be able to stay in the market $\tilde{z}^*$, thus leading some firms out of the market. This *selection effect* triggers a reallocation of resources from the exiting firm to the more productive firms, which are also those innovating at a faster pace. Thus, the selection effect produced by trade liberalization not only raises the level of productivity as in Melitz (2003) but also its growth rate.

As stated before, there is a another more standard channel through which trade-induced increases in competition affects growth. Trade reduces the level of oligopolistic inefficiency in the differentiated goods sector, thus raising the quantity produced of each variety. Since innovation is cost reducing, the marginal benefit from a reduction in costs is increasing with the quantity produced, therefore lower markups trigger higher investment in innovation; this is the *direct competition effect*. This channel does not rely on the presence of heterogeneous firms and, as shown by Licandro and Navas (2007), it operates also in a model with a representative firm. The selection effect instead can be obtained only in an heterogeneous firms framework.

Notice that trade liberalization has an anti-variety effect, it reduces the number of produced and consumed varieties M. This is a consequence of the assumption that there is a perfect overlap between the varieties produced by the two economies. The standard pro-variety effect of trade (e.g. Krugman 1980) could be generated by introducing asymmetry in the set of goods produced by the two countries. However, a model with asymmetric countries would complicate the algebra substantially, without adding much to the main mechanism we want to highlight (the effect of trade-induced selection on innovation and growth).

Proposition 4 *The effect of trade liberalization on selection and growth is decreasing in the number of firms*

Proof. See appendix. ■

The intuition behind this result is that for countries with high levels of product market competition, opening up the economy, or implementing a further trade liberalization, is not going to affect much the already low markup rates.

3.3 Firm selection: the competition channel

The channel through which firms' selection operates in this paper is different from the one in Melitz (2003). In Melitz, selection happens through the effects of trade on the labor market: trade liberalization increases labor demand, this bids up wages and the cost of production, thus forcing the least productive firms to exit the market. In our framework, selection works through the effect of trade on product market competition: the reduction in the markup rate brought about by trade reduces profits and pushes the less productive firms out of the market. In Melitz this channel cannot operate because, under the assumption of monopolistic competition and CES preferences, a larger number of competitors does not affect the elasticity of demand. In our oligopolistic model the market structure is endogenous and trade affects the distribution of surviving firms by raising competition in the product market.[11] Melitz and Ottaviano (2007) endogenize markups by assuming an ad hoc structure of preferences that makes them dependent on the number of firms (varieties), thus introducing a selection effect working through trade-induced increases in product market competition. But this transmission channel is obtained by using special preferences in the monopolistic competitive framework, while in our paper it is produced by the oligopolistic interaction among firms.

4 Quantitative analysis

Coming soon.

5 Conclusion

In this paper we have built a rich but tractable model of trade with heterogeneous firms and cost-reducing innovation, in order to account for a set of findings recently emerged from the empirical analyses of trade liberalization: i) the pro-competitive effect of trade on markups, ii) the selection of the most productive firms, and iii) the positive effect on innovation at the firm level. In our framework, the competition channel is at the roots of the selection and innovation effects

[11] The wage channel of firms selection can be easily introduced in our model by removing the homogeneous good that ties the wage to 1.

of trade liberalization, as all other possible channels (market-size, international technology spillovers, terms of trade) have been excluded from the analysis. The endogenous market structure derives directly from Cournot competition among firms. We have shown that trade liberalization reduces markups, thus forcing the less productive firms out of the market. This selection effect interacts with firms' innovation choice by redistributing resources towards the more productive (more innovative) firms, thereby increasing the aggregate long-run investment in innovation.

The innovation effect of trade highlighted in our model suggests the existence of a new channel of welfare gains from trade that has not been explored in the literature. To keep matters simple we have limited the analysis to the steady-state. A full understanding of the pro-competitive dynamic effects of trade requires the analysis of transitional dynamics, which we view as an interesting task for future research. Finally, studying two perfectly symmetric countries with an identical set of goods, does not allow us to obtain any pro-variety effects of trade. Introducing asymmetric countries is an important step for fully exploring the welfare effects of trade liberalization in our framework.

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A Stationary Distribution

Let $\mu(\tilde{z})$ be the equilibrium productivity density, $g(\tilde{z})$ the equilibrium growth rate and $\epsilon(\tilde{z})$ the net entry rate. Finally, let define $1+\kappa(\tilde{z}) \equiv -\frac{\mu'(\tilde{z})\tilde{z}}{\mu(\tilde{z})}$. Note that κ is constant if $\mu(\tilde{z})$ is Pareto.

Proposition 5 *The density $\mu(\tilde{z})$ is a stationary solution iff $\epsilon(\tilde{z}) = -(1+\kappa(\tilde{z}))\, g(\tilde{z})$ for $\tilde{z} \geq \tilde{z}^*$, $\epsilon(\tilde{z}) + \delta \geq 0$*

Proof. The time t productivity of a variety with productivity $\tilde{z}_s$ at time $s < t$, if still operative, is

$$\tilde{z}_t = \tilde{z}_s e^{\int_s^t g(\tilde{z}_\tau)\, \mathrm{d}\tau},$$

where the exponential term cumulates productivity growth from s to t. Let define the net entry factor $\Delta_t = \mathrm{e}^{\int_s^t \epsilon(\tilde{z}_\tau)\, \mathrm{d}\tau}$, such that

$$\mu_t(\tilde{z}_t) = \mu_s(\tilde{z}_s)\Delta_t,$$

where $\mu_t(\tilde{z}_t)$ is an equilibrium productivity density. By differentiating with respect to t,

$$\mu_t'(\tilde{z}_t)\tilde{z}_t g(\tilde{z}_t) + \frac{\partial \mu_t(.)}{\partial t} = \epsilon(\tilde{z}_t)\, \mu_t(\tilde{z}_t).$$

It's easy to see that

$$\epsilon(\tilde{z}) = \frac{\mu'(\tilde{z})\, \tilde{z}}{\mu(\tilde{z})} g(\tilde{z}) = -(1+\kappa(\tilde{z}))\, g(\tilde{z}) \tag{22}$$

is necessary and sufficient for $\mu(\tilde{z})$ to be stationary, i.e., for $\frac{\partial \mu_t(.)}{\partial t} = 0$. ■

B Firm problem in open economy

Each firm solves the following problem

$$\begin{aligned} V_s &= \max_{\left(q^D_{D,t}, z_{D,t}\right)_s^\infty} \int_s^\infty \left[\left(p_{D,t} - \frac{1}{z_{D,t}}\right) q^D_{D,t} + \left(p_{F,t} - \frac{\tau}{z_{D,t}}\right) q^F_{D,t} - h_{D,t} - \lambda\right] e^{-\int_s^t (r_z+\delta)\,\mathrm{d}z}\,\mathrm{d}t \\ & \text{s.t.} \\ & p_{D,t} = \frac{E_{D,t}}{X^\alpha_{D,t}}\, x^{\alpha-1}_{D,t} \qquad \text{and} \qquad p_{F,t} = \frac{E_{F,t}}{X^\alpha_{F,t}}\, x^{\alpha-1}_{F,t} \\ & x_{D,t} = \hat{x}^D_{D,t} + q^D_{D,t} + x^D_{F,t} \quad \text{and} \quad x_{F,t} = \hat{x}^F_{D,t} + q^F_{D,t} + x^F_{F,t} \\ & \dot{z}_{D,t} = A\,\hat{z}_{D,t} h_{D,t} \\ & z_{D,s} > 0, \end{aligned}$$

where $p_{j,t}$, $E_{j,t}$ and $X^\alpha_{j,t}$ are the domestic price, expenditure and total composite good respectively for country $j = D, F$, and q^j_i is the quantity sold from source country i to destination country j. Writing down the current value Hamiltonian and solving it yields the following first order conditions

$$\left[(\alpha - 1)\,\frac{q^D_{D,t}}{x_{D,t}} + 1\right] p_{D,t} = \frac{1}{(1+b)z_{D,t}} \tag{23}$$

$$\left[(\alpha - 1)\,\frac{q^F_{D,t}}{x_{D,t}} + 1\right] p_{F,t} = \frac{\tau}{(1+b)z_{D,t}} \tag{24}$$

$$1 = \upsilon_{D,t} A \hat{z}_{D,t}, \tag{25}$$

$$\frac{z^{-2}_{D,t}}{\upsilon_{D,t}}\left(q^D_{D,t} + \tau q^F_{D,t}\right) = \frac{-\dot{\upsilon}_{D,t}}{\upsilon_{D,t}} + r_t + \delta, \tag{26}$$

Since the two countries are symmetric, $q^D_{D,t} = q^F_{F,t} \equiv q_t$, $q^F_{D,t} = q^D_{F,t} = \breve{q}_t$, $x_{D,t} = x_{F,t} \equiv x_t$, $E_{D,t} = E_{F,t}$, $X_{D,t} = X_{F,t}$, $p_{D,t} = p_{F,t}$. From (23) and (24) and using $q_t/x_t + \ \breve{q}_t/x_t = 1/n$ yields

$$\left[(\alpha - 1)\,\frac{q_t}{x_t} + 1\right] = \frac{2n - 1 + \alpha}{n\,(1+\tau)} \equiv \theta_D \tag{27}$$

$$\left[(\alpha - 1)\,\frac{\breve{q}_t}{x_t} + 1\right] = \tau\frac{2n - 1 + \alpha}{n\,(1+\tau)} \equiv \theta_F = \tau\theta_D \tag{28}$$

which allows us to rewrite (23) and (24) as follows

$$\theta_D \frac{E_t}{X^\alpha_t}\, x^{\alpha-1}_t = \frac{1}{(1+b)z_t} \quad \text{and} \quad \tau\theta_D \frac{E_t}{X^\alpha_t}\, x^{\alpha-1}_t = \frac{\tau}{(1+b)z_t}.$$

Multiplying the above equations by q_t and $\breve{q}_t$ and summing up we obtain

$$\frac{q_t + \tau\breve{q}_t}{(1+b)z_t} = n\left[\theta_D\frac{q_t}{x_t} + \tau\theta_D\frac{\breve{q}_t}{x_t}\right]\frac{E_t}{n}\left(\frac{x_t}{X_t}\right)^\alpha.$$

Using $x_t = \{[1/(1+b)z_t]\,(X_t^\alpha/\theta_D E_t)\}^{\frac{1}{\alpha-1}}$, it is easy to prove that $(x_t/X_t)^\alpha = \widetilde{z}_t$. From (27) and using $q_t/x_t + \ \breve{q}_t/x_t = 1/n$ we obtain

$$\frac{q_t + \tau\breve{q}_t}{(1+b)z_t} = \theta^T e_t \widetilde{z}_t \tag{29}$$

where $e_t = E_t/n$ and

$$\theta^T = \frac{2n-1+\alpha}{n\,(1+\tau)^2\,(1-\alpha)}\left[\tau^2\,(n+\alpha-1) + n\,(2\tau-1) + 1-\alpha\right]$$

is the inverse of the markup in the open economy.

C Exit in open economy

The productivity cutoff is determined solving the following equation

$$\pi_t(\tilde{z}^*) = \left(p_t - \frac{1}{\tilde{z}_t^*(1+b)}\right) q_t + \left(p_t - \frac{\tau}{\tilde{z}_t^*(1+b)}\right)\breve{q}_t - h_t - \lambda = 0$$

Using $p_t = \frac{1}{\theta_D(1+b)z_t}$ and $h_t = \frac{1}{(1+b)}\theta^T e_t \tilde{z}_t - (\rho+\delta)/A$ obtained from (21), yields

$$\frac{1}{\theta_D}\frac{q_t + \breve{q}_t}{(1+b)\tilde{z}_t^*} - \left(\frac{q_t + \tau\breve{q}_t}{(1+b)\tilde{z}_t^*}\right)\left(1 + \frac{1}{1+b}\right) + \frac{\rho+\delta}{A} - \lambda = 0.$$

With the same procedure used to derive (29) we obtain

$$\frac{q_t + \breve{q}_t}{(1+b)z_t} = \theta_D e_t \widetilde{z}_t$$

which, together with (29), yields

$$\left[1 - \left(\frac{2+b}{1+b}\right)\theta^T\right] e_t \tilde{z}_t^* + \frac{\rho+\delta}{A} - \lambda = 0.$$

This expression is similar to (EC) except for the markup $1/\theta^T$ instead of $1/\theta$.

D Non-linear effect of trade liberalization

Here we show that the competition effect of trade is decreasing in the number of firms n. This can be seen by differentiating θ^T with respect to τ

$$\frac{\partial\theta^T}{\partial\tau} = \frac{-2\,(2n-1+\alpha)}{n\,(1+\tau)^2\,(1-\alpha)}\left[\frac{2n\,(\tau^2+\tau-2) - .\,(\tau^2-1)\,(1-\alpha)}{1+\tau}\right]$$

It is easy to see that this derivative is decreasing in n.