Innovation, Firm Dynamics, and International Trade*

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Abstract

We present a general equilibrium model of the decisions of firms to innovate and to engage in international trade. We use the model to study the changes in aggregate productivity that arise as firms' exit, export, process- and product innovation decisions respond to a change in the marginal cost of international trade. Our central finding is that, despite the fact that a change in trade costs can have a substantial impact on heterogeneous firms' exit, export, and process innovation decisions, the impact of changes in these decisions on aggregate productivity is largely offset by the response of product innovation. Our results suggest that microeconomic evidence on individual firms' responses to changes in international trade costs may not be informative about the macroeconomic implications of changes in these trade costs for aggregate productivity.

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1. Introduction

A large and rapidly growing empirical literature has documented that a reduction in international trade costs can have a substantial impact on individual firms' decisions to exit, export, and invest in research and development both to improve the cost or quality of existing products, and to create new products.[1] Motivated by these observations, we build a simple general equilibrium model to examine the question: Do considerations of the impact of a reduction in trade costs on heterogeneous firms' decisions to exit, export, and innovate lead to new answers to the macroeconomic question about the impact of such cost changes on aggregate productivity? Our answer is, largely, No.

The model we use in coming to this answer follows the recent literature on heterogeneous firms and international trade (e.g. Krugman 1979, Bernard, Eaton, Jensen, and Kortum 2003, Melitz 2003, and Helpman 2006). We model firms as producing differentiated products that are traded internationally subject to both fixed and marginal costs of exporting. Our model of innovation builds on Griliches' (1979) knowledge capital model of firm productivity. In our model, each firm has a stock of a firm-specific factor that determines its current profit opportunities. The model includes two forms of innovation: investment to increase the stock of this firm-specific factor in an existing firm — *process innovation* — and investment to create new firms with a new initial stock of the firm-specific factor — *product innovation*.

We use this model to study the effects of a change in marginal trade costs on aggregate productivity.[2] For this analysis, we decompose the change in aggregate productivity that arises from a change in the marginal costs of trade into two components. The first component is the *direct effect* of a change of trade costs on aggregate productivity, holding fixed firms' exit, export, process, and product innovation decisions. The magnitude of this direct effect is determined simply by the share of exports in production, and hence is independent of the details of our model of heterogeneous firms' decisions. The second component is the *indirect effect* that arises from changes in firms' exit, export, process, and product innovation decisions caused by the change in trade costs.

What determines the magnitude of this indirect effect? A theoretical literature stemming

[1] Bernard, Jensen, Redding and Schott (2007) survey this literature. In addition, see the work of Bustos (2007), De Loecker (2007), Lileeva and Trefler (2007), and Aw, Roberts and Xu (2009).

[2] Throughout this paper, we consider an ideal measure of aggregate productivity that takes into account the introduction of new varieties. This concept of aggregate productivity is not necessarily what is measured in the data (see, for example, Bajona, Gibson, Kehoe, and Ruhl 2008). We focus on the ideal measure of productivity because it is this that matters for welfare in our model.

from the work of Krugman (1979), Grossman and Helpman (1991), and Rivera-Batiz and Romer (1991) has studied this question, focusing only on the impact of a change in international trade costs on firms' decisions to create new product varieties, that is, to engage in product innovation. Our main finding is that our more complex model that also takes into account the heterogeneous responses of firms' exit, export, process and product innovation decisions, leads to largely the same implications for the magnitude of the indirect effect of a reduction in trade costs on aggregate productivity as found in this earlier literature. Whereas when firms are heterogeneous, a change in international trade costs can substantially affect some firms' exit, export, and process innovation decisions, their impact on aggregate productivity is largely offset in general equilibrium by changes in product innovation.

We first present this finding regarding the steady-state impact of a change in marginal trade costs on aggregate productivity as an analytical result for three special cases of our model. In the first special case, we assume that *all firms export.* This specification extends the work of Krugman (1979) by considering firms' exit and process innovation decisions, as well as their product innovation decisions. In the second special case, only the most productive firms export, but firms have *no productivity dynamics* after entry. Hence, this specification corresponds to the Melitz (2003) model. In the third special case, the *exogenous-selection* version of our model, firms have productivity dynamics due to endogenous process innovation, but their exit and export decisions are independent of firm size. In all cases we assume symmetric countries, and in the second and third cases, we also assume that the real interest rate is zero. We show analytically that the indirect effect on aggregate productivity of a change in the marginal costs of trade is, to a first-order approximation, the same in all three of these special cases and equal to the indirect effect found by the earlier models with only product innovation. Hence, for our special cases, the details of how a change in trade costs affects firms' exit, export, and process innovation decisions have no first-order effects on the model's implications for aggregate productivity in the steady state.

We find this result striking because different specifications of our model give rise to very different implications for firms' exit, export, and process innovation decisions at the micro level. In particular, when firms are heterogeneous, a reduction in trade costs leads to a reallocation of production, export status, and investments in process innovation from smaller, less-productive, non-exporting firms to larger, more-productive, exporting firms and this reallocation does lead to a change in the productivity of the average firm. Yet this reallocation does not have a first-order effect for the model's implications for aggregate productivity. Why?

The logic of our result follows from firms' free-entry condition: the profits associated with creating a new product must be zero in equilibrium. Ceteris-paribus, a reduction in international trade costs raises the profits associated with creating a new product. In equilibrium, to satisfy the free-entry condition, this increase in expected profits must be offset by an increase in the real wage and a change in aggregate output, both of them which are determined by aggregate productivity. We prove our result by showing that the change in aggregate productivity required in equilibrium is, to a first-order approximation, independent of the details of firms' exit, export, and process innovation decisions. In our three special cases, the free-entry condition requires that whatever change in the productivity of the average firm that arises from changes in heterogeneous firms' exit, export, and process innovation decisions must be offset by a change in product innovation so as to ensure that the response of aggregate productivity is consistent with equilibrium.

In establishing our analytical results, we make strong assumptions. We also study the indirect effect of changes in trade costs on aggregate productivity when some of these assumptions are relaxed. To do so, however, we must solve the model numerically. We thus consider a parameterized version of our model that accounts for some salient features on the share of exporters in output and employment and the firm size distribution in the U.S. economy. Our quantitative results confirm our analytical findings regarding the first-order effects of a change in marginal trade costs on aggregate productivity, both when the interest rate is low and when firms' investments in process innovation are inelastic to changes in the incentives to innovate.

We find, however, that in a specification of our model with both positive interest rates and elastic process innovation, the changes in firms' process and product innovation decisions are not fully offsetting. This is why we qualify our answer to the question that motivates our work here. However, we find that the response of aggregate productivity due to this indirect effect is at least an order of magnitude smaller than the response of the productivity of the average firm due to changes in firms' process innovation decisions. When we consider the welfare implications of a change in international trade costs with positive interest rates and elastic process innovation, the change in aggregate productivity across steady-states is not associated with a substantial change in welfare relative to the model with inelastic process innovation.

Our model is related to several models in the literature. When our firms' process innovation choices are inelastic, our model is an open economy version of the models of Hopenhayn (1992) and Luttmer (2007a) in which firms experience exogenous random shocks to their productivity.[3] Our model of process innovation is similar to that of Ericson and Pakes (1995), in which the fruits of innovative activity are stochastic. With this assumption, our model can account for simultaneous growth and decline, and entry and exit of firms in steady-state.[4] Our model is also related to the models of Yeaple (2005), Bustos (2007), and Costantini and Melitz (2008); these researchers study the adoption of technology improvements by exporters and non-exporters in response to a change in trade costs.[5] Our result that a change in international trade costs has no impact on innovative effort if all firms export echoes the result of Eaton and Kortum (2001) in a model of quality ladders embedded in a multi-country Ricardian model of international trade. Our work also complements that on firm-level innovation by Klette and Kortum (2004) and Lentz and Mortensen (2006).

Our work here is also related to a large literature on the aggregate implications of trade liberalizations. Baldwin and Robert-Nicoud (2008) study a variant of Melitz's (2003) model that features endogenous growth through spillovers. They show that a reduction in international trade costs can increase or decrease growth by changes in product innovation, depending on the nature of the spillovers and the form of the production function of new goods. Our model abstracts from such spillovers. Arkolakis et al. (2008) show that the welfare gains from a reduction in trade costs in a Melitz (2003) model with Pareto distributed productivities that abstracts from process innovation, are equal to the welfare gains in simpler models of trade that abstract from endogenous exit and export decisions, if these models are calibrated with common trade shares and elasticities.

The paper is organized as follows. Section 2 presents our model, and Section 3 characterizes its symmetric steady-state equilibrium. Section 4 characterizes the steady-state impact of a change in international trade costs in specifications of our model that we can solve for analytically. Section 5 extends the results of Section 4 to specifications that we must solve

[3]Such a model is also considered by Irarrazabal and Opromolla (2008). Furthermore, Arkolakis (2008) extends this model of firm dynamics to account for other salient features of the data on firm dynamics by domestic and exporting firms.

[4]Doraszelski and Jaumandreu (2008) estimate a Griliches' knowledge capital model in which innovative investments within the firm also lead to stochastic productivity improvements.

[5]See also the related work of Navas-Ruiz and Sala (2007), and Long, Raff, and Stähler (2008).

numerically. Section 6 concludes. The Appendix provides proofs and other details for our analytical results.

2. The Model

Time is discrete, and each period is labeled $t = 0, 1, 2, \ldots$. The economy has two countries: home and foreign; variables of the foreign country are denoted with a star. Households in each country are endowed with L units of time.

Production in each country is structured as follows. There is a single, final, nontraded good that can be consumed or used in innovative activities, a continuum of differentiated intermediate goods that are produced and can be internationally traded subject to fixed and variable trade costs, and a nontraded intermediate good that we call the *research good*. This research good is produced using a combination of final output and labor, and is used to pay the costs associated with both process and product innovation, as well as the fixed costs of exporting and production. The productivities of the firms producing the differentiated intermediate goods are determined endogenously through equilibrium process innovation, and the measure of differentiated intermediate goods produced in each country is determined endogenously through product innovation.

Intermediate goods are each produced by heterogeneous firms indexed by two firm-specific state variables, z and n_x, which index the firm's productivity and its fixed costs of exporting, respectively. In what follows, we index the firm's production, pricing, and export decisions by these state variables. We assume that the fixed costs of exporting, n_x, evolve exogenously for each firm according to a Markov process in which the distribution of these costs next period given costs n_x this period, is $\Gamma(n_x'|n_x)$.

A firm in the home country with state variables $s = (z, n_x)$ has productivity equal to $\exp(z)^{1/(\rho-1)}$ and produces output $y_t(s)$ with labor $l_t(s)$ according to the constant returns to scale production technology:[6]

$$y = \exp(z)^{1/(\rho-1)} l. \tag{2.1}$$

In addition, in order to operate, the firm requires fixed costs of n_f units of the research good every period. We rescale firm productivity using the exponent $1/(\rho - 1)$ for expositional

[6] Our model can be easily extended to include other forms of physical and human capital. Consideration of these forms of capital will lead to the standard amplification of the impact of changes in productivity on output.

convenience, where $\rho > 1$. As we explain below, with this rescaling, each firm's equilibrium labor and variable profits are proportional to $\exp(z)$.

The output of a home country firm can be used to produce the home final good, with the quantity of this domestic absorption denoted $a_t(s)$. Alternatively, some of this output can be exported to the foreign country to produce the foreign final good. The quantity of the output of the home firm used in the foreign country is denoted $a_t^*(s)$.

International trade is subject to both fixed and iceberg type costs of exporting. The iceberg type of marginal costs of exporting is denominated in terms of the intermediate good being exported. The firm must export Da^* units of output, with $D \geq 1$, in order to have a^* units of output arrive in the foreign country for use in the production of the foreign final good.

Let $x_t(s) \in \{0, 1\}$ be an indicator of the export decision of home firms with state variables s (with $x_t = 1$ if the firm exports and 0 otherwise). Then, feasibility requires that

$$a_t(s) + x_t(s) D a_t^*(s) = y_t(s) \tag{2.2}$$

and that $x_t(s) n_x$ units of the research good be used to pay the fixed costs of exporting.

A firm in the foreign country with state variables s has the same production technology as the home firm, but with output denoted $y_t^*(s)$, labor $l_t^*(s)$, and domestic absorption $b_t^*(s)$. Exports to the home country, $b_t(s)$, are subject to both fixed and marginal costs; hence, feasibility requires that $x_t^*(s) D b_t(s) + b_t^*(s) = y_t^*(s)$ and that $x_t^*(s) n_x$ units of the foreign research good be used to pay the fixed costs of exporting.

The home final good Y_t is produced from home and foreign intermediate goods with a constant returns to scale production technology of the form

$$Y_t = \left[\int a_t(s)^{1-1/\rho} \, \mathrm{d}M_t + \int x_t^*(s) b_t(s)^{1-1/\rho} \, \mathrm{d}M_t^* \right]^{\rho/(\rho-1)}, \tag{2.3}$$

where M_t is the measure of operating firms in the home country over the state s and M_t^* the corresponding measure in the foreign country. Production of the final good in the foreign country is defined analogously.

The final good in the home country is produced by competitive firms that choose output Y_t and inputs $a_t(s)$ and $b_t(s)$ subject to (2.3), in order to maximize profits while taking as given prices of the final and intermediate goods P_t, $p_{at}(s)$, $p_{bt}(s)$; export decisions $x_t(s)$, $x_t^*(s)$; and measures of operating intermediate good firms M_t, M_t^*. All prices in the home country in period t are stated relative to the price of the research good in that country in the same period, which is normalized to 1. Standard arguments give that equilibrium prices must satisfy

$$P_t = \left[\int p_{at}(s)^{1-\rho}\, \mathrm{d}M_t + \int x_t^*(s) p_{bt}(s)^{1-\rho}\, \mathrm{d}M_t^* \right]^{1/(1-\rho)} \tag{2.4}$$

and are related to quantities by

$$\frac{a_t(s)}{Y_t} = \left(\frac{p_{at}(s)}{P_t} \right)^{-\rho} \text{ and } \frac{b_t(s)}{Y_t} = \left(\frac{p_{bt}(s)}{P_t} \right)^{-\rho}. \tag{2.5}$$

Analogous equations hold for prices and quantities in the foreign country.

The research good in the home country is produced with a constant returns to scale production technology that uses Y_{rt} units of the home final good and L_{rt} units of labor to produce $L_{rt}^{\lambda} Y_{rt}^{1-\lambda}$ units of the research good, with the share of labor in research output denoted by $\lambda \in (0, 1]$. The foreign research good is produced symmetrically. We denote the relative price of the research good across countries by W_{rt}^*. In each country, the research good is produced by competitive firms. Standard cost minimization requires that

$$\frac{\lambda}{1-\lambda} \frac{Y_{rt}}{L_{rt}} = \frac{W_t}{P_t},\ \frac{\lambda}{1-\lambda} \frac{Y_{rt}^*}{L_{rt}^*} = \frac{W_t^*}{P_t^*}, \tag{2.6}$$

and that, given our choice of numeraire,

$$1 = \lambda^{-\lambda} (1-\lambda)^{-(1-\lambda)} (W_t)^{\lambda} (P_t)^{1-\lambda} \text{ and } W_{rt}^* = \lambda^{-\lambda} (1-\lambda)^{-(1-\lambda)} (W_t^*)^{\lambda} (P_t^*)^{1-\lambda}. \tag{2.7}$$

Here, W_t (or W_t^*) denotes the wage for workers in the home (or foreign) country.

Intermediate good firms in each country are monopolistically competitive. A home firm with state variables s faces a static profit maximization problem of choosing labor input $l_t(s)$, prices $p_{at}(s)$, $p_{at}^*(s)$, quantities $a_t(s)$, $a_t^*(s)$, and whether or not to export $x_t(s)$, in order to maximize current period profits, taking as given the wage rate W_t, and prices and output of the final good in both countries P_t, P_t^*, Y_t, and Y_t^*. This profit maximization problem is written as

$$\Pi_t(s) = \max_{y,l,p_a,p_a^*,a,a^*,x \in \{0,1\}} p_a a + x p_a^* a^* - W_t l - x n_x \tag{2.8}$$

subject to (2.1), (2.2), and (2.5).

Productivity at the firm level evolves over time depending on the firm's investments in improving its productivity and on idiosyncratic productivity shocks. We model this evolution as follows. At the beginning of each period t, every existing firm has a probability δ of exiting exogenously and a probability $1-\delta$ of surviving to produce. Surviving firms can choose either to exit or to continue to operate and pay the fixed costs of operation n_f in terms of the research good. A continuing firm with state s that invests $\exp(z)\,c(q)$ units of the research good in improving its productivity in the current period t has a probability q of having productivity $\exp(z+\Delta_z)^{1/(\rho-1)}$ and a probability $1-q$ of having productivity $\exp(z-\Delta_z)^{1/(\rho-1)}$ in the next period $t+1$. We refer to the firm's choice of q as its *process innovation decision*, and to the firm's expenditure of $\exp(z)c(q)$ units of the research good as its *investment in process innovation*. We assume that $c(q)$ is increasing and convex in q.[7]

With this evolution of firm productivity, the expected, discounted present value of profits for a firm with state variables s satisfies a Bellman equation:

$$V_t(z,n_x) = \max\left[0, V_t^o(z,n_x)\right] \tag{2.9}$$

$$V_t^o(z,n_x) = \max_{q\in[0,1]} \Pi_t(z,n_x) - \exp(z)c(q) - n_f \tag{2.10}$$

$$+(1-\delta)\frac{1}{R_t}\int_{n_x'}\left[qV_{t+1}(z+\Delta_z,n_x') + (1-q)V_{t+1}(z-\Delta_z,n_x')\right]\mathrm{d}\Gamma(n_x'|n_x),$$

where $\Pi_t(s)$ is given by (2.8) and R_t is the world interest rate in period t (in units of the home research good). Note that here we express this Bellman equation for the firm's expected, discounted present value of profits $V_t(s)$ in units of the research good. We find this convention useful in characterizing equilibrium. We let $q_t(s)$ denote the optimal process innovation decision of the firm in the problem (2.10).

Since for each value of n_x the value function of operating firms $V_t^0(z,n_x)$ is strictly increasing in z, clearly, in each period t, the decision of firms to operate (2.9) follows a cutoff rule, with firms with productivity at or above a cutoff $\bar{z}_t(n_x)$ choosing to operate and firms with productivity below that cutoff exiting. Note that if $n_f=0$, then $V_t^o(s)=V_t(s)$ and $\bar{z}_t(n_x)=-\infty$; hence, there is no endogenous exit.

New firms are created with an investment of the research good. Investment of n_e units of the research good in period t yields a new firm in period $t+1$, with initial state variables s drawn from a distribution G. In any period in which new firms enter, free entry requires

[7] With this scaling of the innovation cost function, $\exp(z)$, we are assuming that the process innovation cost required to increase the size of the firm by a fixed percentage scales with the size of the firm. This will imply that, for sufficiently large firms, their growth rate is independent of size, consistent with Gibrat's law. Note also that if the time period is small, then our binomial productivity process approximates a geometric Brownian motion in continuous time, as in the work of Luttmer (2007a). Our model differs from Luttmer's in that our firms control the drift of this process through investment of the research good.

that

$$n_e = \frac{1}{R_t} \int V_{t+1}(s)\ \mathrm{d}G. \tag{2.11}$$

Note that both sides of this equation are expressed in units of the research good. Let M_{et} denote the measure of new firms entering in period t that start producing in period $t+1$. The analogous Bellman equation holds for the foreign firms as well. We refer to M_{et} as the *product innovation* decision because this is the mechanism through which new differentiated products are produced.

Households in the home country have preferences of the form $\sum_{t=0}^{\infty} \beta^t \log(C_t)$, where C_t is their consumption of the home final good in period t, and $\beta \leq 1$ is their discount factor. Households in the foreign country have preferences of the same form over consumption of the foreign final good C_t^*. Each household in the home country faces an intertemporal budget constraint of the form

$$P_0 C_0 - W_0 L + \sum_{t=1}^{\infty} \left(\prod_{j=1}^{t} \frac{1}{R_j} \right) (P_t C_t - W_t L) \leq \bar{W}, \tag{2.12}$$

where $\bar{W}$ is the value of the initial stock of assets held by the household. Households in the foreign country face similar budget constraints with wages, prices, and assets all labelled with stars.

Feasibility requires that for the final good,

$$C_t + Y_{rt} = Y_t \tag{2.13}$$

in the home country, and the analogous constraint holds in the foreign country. The feasibility constraint on labor in the home country is given by

$$\int l_t(s)\ \mathrm{d}M_t + L_{rt} = L, \tag{2.14}$$

where $\int l_t(s)\ dM_t$ denotes total employment in the production of intermediate goods and L_{rt} denotes employment in the production of the research good and likewise in the foreign country.

The feasibility constraint on the research good in the home country is

$$M_{et} n_e + \int \left[n_f + x_t\left(s\right) n_x + \exp(z) c(q_t(s)) \right] \mathrm{d}M_t = L_{rt}^{\lambda} Y_{rt}^{1-\lambda} \tag{2.15}$$

and likewise in the foreign country.

The evolution of the measure of operating firms M_t over time is given by the exogenous probability of exit δ, the decisions of operating firms to invest in their productivity $q_t(s)$, and the measure of entering firms in period t, M_{et}. The measure of operating firms in the home country in period $t+1$ with state variables less than or equal to $s' = (z', n'_x)$, denoted by $M_{t+1}(z', n'_x)$, is equal to the sum of three inflows of firms: new firms founded in period t, firms continuing from period t that draw positive productivity shocks (and, hence, had productivities lower than $z' - \Delta_z$ in period t), and firms continuing from period t that draw negative productivity shocks (and, hence, had productivities lower than $z' + \Delta_z$ in period t). We write this as follows:

For $z' \geq \bar{z}'_{t+1}(\tilde{n}'_x)$,

$$M_{t+1}(z', n'_x) = M_{et}\left[G(z', n'_x) - G(\bar{z}'_{t+1}(n'_x), n'_x)\right] \tag{2.16}$$

$$+ (1-\delta)\int_0^{n'_x}\left[\int_{-\infty}^{z'-\Delta_z}\int_{\{n_x\}} q_t(z, n_x)\ \mathrm{d}M_t(z, n_x)\right]\mathrm{d}\Gamma(\tilde{n}'_x|n_x)$$

$$+ (1-\delta)\int_0^{n'_x}\left[\int_{\bar{z}'_{t+1}(\tilde{n}'_x)}^{z'+\Delta_z}\int_{\{n_x\}} \big(1 - q_t(z, n_x)\big)\mathrm{d}M_t(z, n_x)\right]\mathrm{d}\Gamma(\tilde{n}'_x|n_x).$$

For $z' < \bar{z}'_{t+1}(\tilde{n}'_x)$, $M_{t+1}(z', n'_x) = 0$. The evolution of $M^*_t(z)$ for foreign firms is defined analogously.

We assume that the households in each country own those firms that initially exist in period 0. Thus, we require that the initial assets of the households in both countries sum to the total value of these firms:

$$\bar{W} + \bar{W}^* = \int V_0(s)\ \mathrm{d}M_0 + \int V^*_0(s)\ \mathrm{d}M^*_0. \tag{2.17}$$

An equilibrium in this economy is a collection of sequences of aggregate prices and wages $\{R_t, P_t, P^*_t,\ W_t, W^*_t, W^*_{rt}\}$ and prices for intermediate goods $\{p_{at}(s),\ p^*_{at}(s),\ p_{bt}(s),\ p^*_{bt}(s)\}$, a collection of sequences of aggregate quantities $\{Y_t,\ Y^*_t,\ C_t,\ C^*_t,\ Y_{rt},\ Y^*_{rt},\ L_{rt},\ L^*_{rt}\}$ and quantities of the intermediate goods $\{a_t(s),\ a^*_t(s),\ b_t(s),\ b^*_t(s),\ l_t(s),\ l^*_t(s)\}$, initial assets $\bar{W}$, $\bar{W}^*$, and a collection of sequences of firm value functions and profit, exit, export, and process innovation decisions $\{V_t(s),\ V^*_t(s),\ V^o_t(s),\ V^{o*}_t(s),\ \Pi_t(s),\ \Pi^*_t(s),\ \bar{z}(n_x),\ \bar{z}^*(n_x),\ x_t(s),\ x^*_t(s),\ q_t(s),\ q^*_t(s)\}$ together with measures of operating and entering firms $\{M_t,\ M_{et},\ M^*_t,\ M^*_{et}\}$ such that households in each country maximize their utility subject to their budget constraints, intermediate good firms in each country maximize within-period profits, final good firms in each country maximize profits, all of the feasibility constraints are satisfied, and the measures of operating firms evolve as described above.

In most of our analysis, we focus our attention on equilibria that are *symmetric* in two basic ways. We assume that the distribution of initial assets is such that expenditures are equal across countries in period 0 and, hence, in every period. We also assume that each country starts with the same distribution of operating firms by productivity and, hence, because prices and wages are equal across countries, continue to have the same distribution of operating firms by productivity in each subsequent period. In such a symmetric equilibrium, we have $Y_t = Y_t^*$, $P_t = P_t^*$, $W_t/P_t = W_t^*/P_t^*$, and $W_{rt}^* = 1$.

A *steady state* of our model is an equilibrium in which all of the variables are constant. A *symmetric steady state* is an equilibrium that is both symmetric in our sense and a steady state. In what follows, we omit time subscripts when discussing steady states.

3. The Symmetric Steady State

Now we present the equations that characterize a symmetric steady state equilibrium in our model. We first characterize the firms' pricing, exit, export, and process innovation decisions. We show that these decisions are the solution to a one-dimensional fixed-point problem. We then characterize the aggregate quantities and prices, taking as given the firms' decisions. Finally, we present a central result: In the steady state, the combined impact of firms' exit, export, and process and product innovation decisions on aggregate productivity must offset each in order to keep firms' profits consistent with free entry.

3.1. Firm Decisions

Consider the static profit maximization problem (2.8) for an operating firm in the home country. All operating firms choose a constant markup over their marginal costs, so that equilibrium prices are given by

$$p_a(s) = \frac{\rho}{\rho - 1}\frac{W}{\exp(z)^{1/(\rho-1)}}, \text{ and } p_a^*(s) = \frac{\rho}{\rho - 1}\frac{DW}{\exp(z)^{1/(\rho-1)}}.$$

Given the demand of final good firms for intermediate inputs (2.5), home intermediate firms with state variables s have variable profits on their home sales in terms of the numeraire, $\Pi_d \exp(z)$, with the constant on variable profits Π_d given by

$$\Pi_d = \frac{(W/P)^{1-\rho} PY}{\rho^\rho (\rho - 1)^{1-\rho}}\ , \tag{3.1}$$

and variable profits $\Pi_x \exp(z)$ on their foreign sales, with $\Pi_x = \Pi_d D^{1-\rho}$. As is standard, domestic variable profits are decreasing in the real wage W/P, increasing in the price charged by other firms P, and increasing in the scale of final good production Y.

Total static profits are

$$\Pi(s) = \Pi_d \exp(z) + \max(\Pi_x \exp(z) - n_x, 0)\,. \tag{3.2}$$

We now characterize the firms' exit, export, and process innovation decisions as the unique solution of a one-dimensional fixed-point problem. We solve for a fixed-point over the constant Π_d in firms' variable profits, as defined in (3.1).

To do so, consider first firms' export decisions, $x(s)$. Given a value of Π_d, these decisions are determined by the static condition that variable profits from exports must exceed fixed costs of exporting, or

$$x(z, n_x) = 1 \text{ if and only if } \Pi_d D^{1-\rho} \exp(z) \geq n_x. \tag{3.3}$$

To solve for firms' steady-state exit and process innovation decisions, we must solve the firms' Bellman equation, (2.9), removing the time subscripts from all variables and letting $R_t = 1/\beta$. Standard arguments give that this Bellman equation has a unique solution $V(s)$, corresponding to any given value of Π_d under appropriate parameter restrictions.[8] In addition, the solution for $V(s)$ is weakly increasing in Π_d, while the value function of operating firms, $V^o(s)$, is strictly increasing in Π_d.

We use the free-entry condition (2.11) to solve for the equilibrium value of Π_d. To see that a unique solution for Π_d exists, first observe that the right side of the free-entry condition (2.11) is weakly increasing in Π_d and that if it is strictly positive (when a positive mass of newly entering firms choose to operate), then it is also strictly increasing in Π_d. Second, note that the right side of (2.11) is equal to zero when $\Pi_d = 0$ and becomes arbitrarily large

[8] The parameter restrictions required ensure that the net present value of firms' profits remain bounded for any choice of process innovation. A strong sufficient condition is that $\beta(1-\delta)\exp(\Delta_z) < 1$. When numerically solving our model, we check the following weaker sufficient conditions: For all $q \in [0,1]$ such that $\beta(1-\delta)[q\exp(\Delta_z) + (1-q)\exp(\Delta_z)] \geq 1$, we need $\Pi_d\left(1 + D^{1-\rho}\right) - c(q) < 0$. The interpretation of this condition is that if it is possible for a firm to choose process innovation so that variable profits grow faster than the interest rate, then the variable profits associated with this process innovation decision are negative.

as Π_d gets large. Since the fixed costs of entry are strictly positive, there is a unique solution for Π_d.

The solution to this problem now gives us firms' exit decisions $\bar{z}(n_x)$, export decisions $x(s)$, and process innovation decisions $q(s)$. These decisions, under certain parameter restrictions, imply from (2.16) a steady-state distribution of state variables across firms scaled by the mass of entering firms, $\tilde{M}(s) = M(s)/M_e$. The parameter restrictions required imply that the equilibrium process innovation decision of large firms leads them to shrink in expectation.[9]

3.2. Aggregate Quantities and Prices

Now assume that the firms' exit, export, and process innovation decisions are given and lead to a steady-state scaled distribution across states, $\tilde{M}(s)$. To solve for aggregate quantities and prices, we define two indices of aggregate productivity across firms implied by firms' decisions,

$$\begin{aligned} Z_d &= \int [1 - x(z, n_x)] \exp(z) \, \mathrm{d}\tilde{M}(z, n_x), \text{ and} \\ Z_x &= \int x(z, n_x) \exp(z) \, \mathrm{d}\tilde{M}(z, n_x). \end{aligned} \tag{3.4}$$

The first of these, Z_d, is an index of productivity aggregated across all operating, non-exporting home firms, and the second, Z_x, is an index of productivity aggregated across all home firms that export, both scaled by the mass of entering firms. In a symmetric steady state, Z_x is also an index of productivity aggregated across all foreign firms that export to the home country.

From the firm's static profit maximization problem (2.8), we have that the production employment of home firms in a symmetric steady state is given by

$$l(s) = \left(\frac{\rho - 1}{\rho}\right)^{\rho} \left(\frac{W}{P}\right)^{-\rho} Y \exp(z) \left[1 + x(s) D^{1-\rho}\right]. \tag{3.5}$$

[9] To check that a stationary distribution exists, one must check that the equilibrium process innovation decisions satisfy

$$\lim_{z \to \infty} (1 - \delta) \{q(z, n_x) \exp(\Delta_z) + [1 - q(z, n_x)] \exp(-\Delta_z)\} < 1$$

for all values of n_x. If this condition is violated, it is possible to have an equilibrium with endogenous growth through process innovation in which there is no product innovation. Our assumption that $\lambda > 0$ and that there are no spillovers guarantees that our model does not have endogenous growth through product innovation. If $\lambda = 0$, then our model corresponds to the "lab equipment model" and it can have endogenous growth through product innovation.

Given that firm revenues are proportional to firm employment, the share of exports in the value of production of intermediate inputs is given by

$$s_x = \frac{Z_x D^{1-\rho}}{Z_d + (1 + D^{1-\rho}) Z_x}. \tag{3.6}$$

Note that the share of total production employment accounted for by exporters is $s_x \left(1 + D^{1-\rho}\right) / D^{1-\rho}$.

We compute the average expenditures on the research good per entering firm, which we denote by Υ, with

$$\Upsilon = n_e + \int \left[n_f + x(s)\, n_x + \exp(z) c(q(s))\right] \mathrm{d}\tilde{M}(z, n_x). \tag{3.7}$$

Given Π_d, Z_d, Z_x, and Υ, the symmetric steady-state values of W/P, Y, L_r, Y_r, M_e, and C solve the following six equations: (2.6),

$$\frac{W}{P} = \frac{\rho - 1}{\rho} \left[M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)\right]^{1/(\rho-1)}, \tag{3.8}$$

$$Y = \left[M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)\right]^{1/(\rho-1)} (L - L_r), \tag{3.9}$$

$$L_r = \frac{\lambda}{\lambda + \zeta(\rho - 1)} L, \tag{3.10}$$

$$\Pi_d = \frac{\lambda^\lambda (1 - \lambda)^{1-\lambda}}{\rho^\rho (\rho - 1)^{1-\rho}} (W/P)^{1-\rho-\lambda} Y, \text{ and} \tag{3.11}$$

$$C = Y \left[1 - \frac{(1 - \lambda)}{\zeta \rho}\right], \tag{3.12}$$

where $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon$ is the ratio of total variable profits to total expenditures on the research good. We derive these equations in the Appendix.

Since labor is the only variable factor of production, aggregate productivity from equation (3.9) is given by

$$Z = \left[M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)\right]^{1/(\rho-1)}. \tag{3.13}$$

3.3. The Aggregate Allocation of Labor

In solving our model, we use the following two lemmas regarding the aggregate allocation of employment and the ratio of consumption to final output. Lemma 1 states that these two variables change with a change in marginal trade costs only if the ratio of total variable profits to total expenditures on the research good also changes. We show in Lemma 2 that if the interest rate is zero ($\beta = 1$), then the aggregate allocation of labor and the ratio of consumption to final output are independent of marginal trade costs.

Lemma 1: The steady-state allocation of labor to produce the research good, L_r, and the steady-state ratio of consumption to output, C/Y, are functions of only the ratio of total variable profits to total expenditures on the research good, ζ, and the parameters λ, ρ, and L.

Proof: See expressions (3.10) and (3.12).

Lemma 2: With $\beta = 1$, in the steady state, average variable profits across firms equal average expenditures across firms on the research good, so $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho} \right) \right] / \Upsilon = 1$. Hence, by (3.10), L_r is a constant fraction of the labor force given by $L_r = \left[\lambda / \left(\lambda + \rho - 1 \right) \right] L$, independent of the trade cost D.

Proof: Free entry requires that for an entering firm, the expected present value of variable profits equals the expected present value of expenditures on the research good. In a steady state in which the interest rate is zero, these expected present values are equal to their cross-sectional averages across firms. More details are given in the Appendix.

In our model, households' utility is not defined when $\beta = 1$. We interpret Lemma 2 as a useful limiting result as the discount factor approaches 1.

3.4. A Recursive Algorithm to Solve the Steady State

Together, these two Lemmas give us the following algorithm to solve for a symmetric steady state of the model as a function of the marginal trade cost D. First, we use the free-entry condition (2.11) to solve for the equilibrium value of Π_d. Associated with the equilibrium value of Π_d are firms' exit, export, and process innovation decisions, which determine the aggregate productivity indices Z_x and Z_d, as well as the average expenditures per entering firm on the research good Υ. Then, we use (3.10) to compute L_r and (3.8), (3.9), and (3.11) to solve for the equilibrium product innovation M_e. Expressions (3.9) and (3.12) then determine output and consumption.

With this algorithm, we see that our model has a certain recursive structure. In equilibrium, the free-entry condition pins down firms' exit, export, and process innovation decisions as well as the aggregate allocation of labor between production employment and research. Product innovation then adjusts to satisfy the remaining equilibrium conditions.[10]

We will use this recursive structure of our model to analyze the impact of a change in

[10] This recursive structure relies on our assumption that all innovation activities use the same research good. If different inputs were required for product and process innovation, then a change in trade costs might affect the relative price of the inputs into these activities and, thus, affect equilibrium process innovation. In this case, the full model must be solved simultaneously.

marginal trade costs on the steady-state equilibrium levels of aggregate productivity, output, and welfare. From (3.13), we know that aggregate productivity is determined by the exit, export, and process and product innovation decisions of firms. A central result of this work is that, in the steady state, the impacts of these decisions on aggregate productivity must offset each other in order for firms' profits to be consistent with free entry. In particular, from the steady-state equilibrium conditions, (3.8), (3.9), and (3.11), we have that

$$\Delta \log \Pi_d = (2 - \rho - \lambda) \Delta \log Z + \Delta \log (L - L_r), \tag{3.14}$$

where Δ denotes the total derivative of a variable.

The intuition for (3.14) is as follows. The free-entry condition, as captured by our Bellman equation, pins down how the variable profits earned by a firm with a given productivity level must change in response to a change in marginal trade costs. With (3.1), this changes in variable profits also pins down the change in the real wage and aggregate output that must occur in the new steady state. Since the real wage and aggregate output are determined by aggregate productivity and the aggregate allocation of labor, we have that the free-entry condition for firms pins down how aggregate productivity and the aggregate allocation of labor must respond to a change in marginal trade costs.

The economics of the coefficient on aggregate productivity in (3.14) is as follows. An increase in aggregate productivity raises the real wage and output one-for-one and decreases the price of the final good in terms of the research good at the rate λ. From (3.1), we know that the combined effect of an increase in aggregate productivity on the constant on variable profits is given by $(2 - \rho - \lambda)$; hence, that term is the appropriate coefficient.

In what follows, we impose the parameter restriction $\rho + \lambda > 2$ so that an increase in aggregate productivity lowers the constant on variable profits. When this restriction is violated, choosing an unbounded level of entry M_e and consumption C in the steady state is socially optimal. To see this, consider a planner seeking to choose Y_r and M_e in order to maximize $C = Y - Y_r$, with the levels of Z_x, Z_d, Υ, and L_r held fixed. Using (2.15) to solve for Y_r in terms of M_e, allows us to state the objective in this problem as $\kappa M_e^{\frac{1}{\rho-1}} - M_e^{\frac{1}{1-\lambda}}$, with $\kappa > 0$. This function is concave in M_e and, hence, has an interior maximum if and only if $\rho + \lambda > 2$. Therefore, when this condition is violated, setting $M_e = \infty$ is optimal. We rule out this parameter configuration because an economy with feasible unbounded consumption in the steady state is uninteresting.[11]

[11] Given the parameter assumption that $\rho + \lambda > 2$, we can show that the social planner chooses exit,

4. Trade Costs and Aggregate Productivity: Analytical Results

In this section, we analytically study the impact of a change in marginal trade costs on aggregate productivity for three special cases of our model. In the first special case, we assume that *all firms export*. In the second special case, only the most productive firms export, but firms have *no productivity dynamics* after entry; hence, this special case of our model corresponds to the model of Melitz (2003). In the third special case, that we refer to as the *exogenous-selection* version of our model, firms have endogenous productivity dynamics from process innovation, but firms' exit and export decisions are independent of size. In the second and third special cases, we also assume that the real interest rate is zero. We show here that a change in those trade costs has the same impact on steady-state productivity, to a first-order approximation, in all three special cases.

To a first-order approximation, a change in marginal international trade costs D has two types of effects on aggregate productivity. One effect is direct; productivity changes only because of the change in trade costs, with firms' exit, export, process, and product innovation decisions held fixed. The other effect of a trade cost change is *indirect effect*; it arises from changes in these decisions, which are themselves responding to the trade cost change. More formally, from equation (3.13), the change in aggregate productivity from a change in trade costs is

$$\Delta \log Z = \underbrace{-s_x \Delta \log D}_{\textbf{Direct Effect}} \tag{4.1}$$

$$+\underbrace{\frac{1}{\rho-1}\left[s_x \frac{1+D^{1-\rho}}{D^{1-\rho}}\Delta \log Z_x + \left(1 - s_x \frac{1+D^{1-\rho}}{D^{1-\rho}}\right)\Delta \log Z_d + \Delta \log M_e\right]}_{\textbf{Indirect Effect}}.$$

The indirect effect of a change in trade costs on aggregate productivity itself has two components. The first component (that is, the sum of the first two terms in brackets) is the indirect effect of a change in trade costs on the productivity of the average firm. The second component, given by $\Delta \log M_e / (\rho - 1)$, is the indirect effect that arises from product innovation, or the creation of new firms.

To calculate the indirect effect on aggregate productivity, we proceed as follows. The

export, and process innovation decisions in the steady state equal to those chosen in equilibrium. Moreover, the optimal and equilibrium steady-state allocations are identical if $\lambda = 1$, and the optimal levels of output, consumption, and product innovation are higher than the equilibrium level of these variables when $\lambda < 1$. The intuition for this result is that the equilibrium monopoly distortion alters the value of entry relative to the cost of entry.

expression (3.14) can be written as

$$\Delta \log \Pi_d = (2 - \rho - \lambda) \times (\text{Direct Effect} + \text{Indirect Effect}) + \Delta \log (L - L_r). \quad (4.2)$$

For our three special cases, we show below that from the Bellman equation, we know that the steady-state change in the constant in variable profits that is consistent with free entry is given by

$$\Delta \log \Pi_d = (\rho - 1) s_x \Delta \log D = (1 - \rho) \times (\text{Direct Effect}). \quad (4.3)$$

When all firms export, or when the interest rate is zero, the steady-state aggregate allocation of labor is unchanged with D, so that $\Delta \log (L - L_r) = 0$. (See Lemmas 1 and 2 for the case in which $\beta = 1$.) Plugging these results into (4.2) gives that the ratio of the indirect effect to the direct effect of a change in trade costs on aggregate productivity is given by

$$\frac{\text{Indirect Effect}}{\text{Direct Effect}} = \frac{1 - \lambda}{\rho + \lambda - 2}. \quad (4.4)$$

This expression (4.4) is a straightforward implication of a standard model of trade with homogeneous firms and monopolistic competition, no productivity dynamics, no fixed costs of production or exporting, and no spillovers, such as the model described by Krugman (1979).

Our main result is that (4.4) characterizes the relative size of the indirect and direct effects in all three special cases of our model. This result has two important implications:

- If $\lambda = 1$, so that the research good is produced entirely with labor, then there is no indirect effect. Hence, the steady-state change in productivity, to a first-order approximation, is simply the direct effect. This means that in equilibrium, the changes in productivity induced by changes in firms' exit, export, process, and product innovation decisions (that is, the indirect effect) must entirely offset each other, to a first-order approximation, in the new steady state.

- Under the more general assumption that $\lambda < 1$, the indirect effect on productivity has the same magnitude, to a first-order approximation, regardless of endogenous process innovation and endogenous or exogenous choices by firms to export and exit.

Later, we explore the extent to which this analytical result holds in more general cases of our model.

When computing the welfare effects of a change in marginal trade costs D, we must consider the impact of this change on consumption in the steady state and its transition dynamics. In Lemma 1, we have proven that the change in the ratio of consumption to output in the steady-state is determined by the same factor ζ that determines the aggregate allocation of labor, L_r. Since in all three special cases of our model, ζ remains constant, we have that steady-state consumption moves one-for-one with steady-state output and that the steady-state change in aggregate output is equal to the change in aggregate productivity. The transition dynamics are computed numerically in Section 5. However, here, at the end of Section 4, we discuss why, if the steady-state effects of a change in marginal trade costs are large, then the transition dynamics are slow.

The line of argument we use here to analyze the direct and indirect effects arising from a change in trade costs does not extend naturally to the analysis of a change in import tariffs that are rebated to a household. A change in tariffs does not entail the same direct effect as a change in trade costs because it does not change the resources consumed in international trade. It is possible, however, to show that, to a first order approximation, the response of aggregate productivity to a change in tariffs is the same in all three special cases of our model if tariffs are initially zero.

4.1. All Firms Export

We start the analysis of this case by establishing, in Proposition 1, that in an economy with no fixed costs of international trade, changes in the marginal costs of trade have no impact at all on the incentives of firms in the steady state to engage in process innovation. We then use this proposition to show that in response to a change in marginal trade costs, $\Delta \log Z_x = \Delta \log Z_d = 0$ and that the change on the constant in variable profits is given by (4.3). We then show, in Proposition 2, that the aggregate allocation of labor is unchanged and that the ratio of indirect to direct effects of changes in marginal trade costs on aggregate productivity is given by (4.4).

Proposition 1: Consider a world economy with no fixed costs of trade ($n_x = 0$). In this economy, a change in marginal trade costs D, has no impact on the steady-state process innovation decisions of firms, $q(s)$.

Proof: We first prove this proposition under the assumption that the economy is in a symmetric steady-state equilibrium. With $n_x = 0$ for all firms, (3.3) implies that all firms export, and the variable profits of a firm with productivity z are $\Pi_d(1 + D^{1-\rho}) \exp(z)$.

Hence, under the assumption that all firms export, the Bellman equation in the steady state, (2.9), can be written with $\tilde{\Pi} \exp(z)$ replacing $\Pi_t(s)$, where $\tilde{\Pi} = \Pi_d(1 + D^{1-\rho})$. Our arguments in the previous section imply that a unique level of $\tilde{\Pi}$ exists which satisfies the free-entry condition (2.11), independent of the parameter D. The corresponding process innovation decisions that solve the Bellman equation at this level of $\tilde{\Pi}$ are the equilibrium exit and process innovation decisions. These are also independent of D.

In a steady-state equilibrium that is not symmetric, the appropriate definition of $\tilde{\Pi}$ is $\Pi_d + \Pi_x D^{1-\rho}$, and the same logic applies. Clearly, the analogous results hold for foreign firms. Q.E.D.

Proposition 1 holds because, in an economy in which all firms export, the increased incentives to innovate resulting from the increase in profits that comes from a reduction in marginal trade costs affect all firms proportionally. The free-entry condition then requires that the increase in profits be exactly offset by an increase in the costs of the research good necessary for innovation. Recalling that we have normalized the price of the research good to 1, we see that this is the intuition for the result that $\tilde{\Pi} = \Pi_d(1 + D^{1-\rho})$ remains unchanged. So, too, does the optimal process innovation decision of all firms.[12]

Proposition 2: Consider a world economy with no fixed costs of trade ($n_x = 0$). In this economy, in response to a change in marginal costs of trade D, the aggregate labor allocation L_r is unchanged, and the ratio of the indirect effect to the direct effect is given by (4.4). This indirect effect corresponds entirely to a change in product innovation.

Proof: We prove this proposition by calculating the terms in (4.2). From Proposition 1, we know that $\Delta \log \Pi_d = -\Delta \log(1 + D^{1-\rho})$. Since all firms in this economy export, the share of exports in intermediate goods' output is equal to the export intensity of each firm, which is given by $D^{1-\rho}/(1 + D^{1-\rho})$. This gives (4.3). An immediate corollary of Proposition 1 is that the firms' exit decision are also unchanged. Hence, the scaled distribution of firms across states, $\tilde{M}(s)$, the productivity indices, Z_d and Z_x, and the ratio of total variable profits to total expenditures on research goods, $\zeta = \Pi_d [Z_d + Z_x(1 + D^{1-\rho})] / \Upsilon$, remain unchanged. From Lemma 1, L_r is also unchanged. Our result follows from expression (4.2). Q.E.D.

Our proof follows from (4.2). That expression is a first-order approximation of the change

[12]Given this intuition, in our model when all firms export, firm-level process innovation decisions are also unaffected by a country moving from autarky to free trade or by changes in tariffs or tax rates on firm profits, revenues, or factor use that alter the variable profit function in the same weakly separable manner with z. Proposition 1 would also hold in a two-sector model in which the aggregate outputs of each sector are imperfect substitutes, and firms face separate entry conditions of the form (2.11).

in steady-state profits (3.14); however, the result can be extended to the full non-linear model. Note also that if $\lambda = 1$, product innovation is unchanged with a change in trade costs. In this case, it is possible to prove Proposition 2 without the use of the free-entry condition, but instead fixing the number of firms in each country. One does require the free-entry condition to prove our result when $\lambda < 1$.

4.2. No Productivity Dynamics

Now, consider a version of our model with fixed operating and export costs, which assumes that $\Delta_z = 0$ and with a time-invariant value of n_x, so that it has no dynamics of firm productivity or export decisions of active firms. In this version of our model, firms choose not to engage in process innovation; hence, this model corresponds Melitz's (2003). In proving the next proposition 3, we establish that the ratio of indirect to direct effects on aggregate productivity from a change in marginal trade costs is given by (4.4) for this version of our model as well.

Proposition 3: In a symmetric steady-state equilibrium of our model with $\Delta_z = 0$, a time-invariant value of n_x, and $\beta = 1$, to a first-order approximation, the ratio of the indirect effect to the direct effect of a change in marginal trade costs D on aggregate productivity is given by (4.4).

Proof: Because $\beta = 1$, Lemma 2 applies in this version of our model, so L_r remains unchanged when marginal trade costs change. Because this model has no dynamics in productivity or export decisions, active firms' value functions in the steady-state equilibrium are given by

$$V(z, n_x) = \frac{1}{\delta} \max \left[0, \Pi_d \exp(z) - n_f + \max\left\{0, \Pi_d \exp(z) D^{1-\rho} - n_x\right\}\right]. \qquad (4.5)$$

The free-entry condition is still (2.11). Because continuing without profits has no option value, firms exit if they draw an initial productivity z which yields a firm's static profits in the domestic market less than zero, $\Pi_d \exp(z) < n_f$. Likewise, firms choose to export only if the static profits associated with doing so are positive, $\Pi_d \exp(z) D^{1-\rho} > n_x$. Using these results to differentiate the free-entry condition (2.11) gives (4.3). The details of this derivation are provided in the Appendix. Proposition 3 is obtained from plugging this last expression into (4.2). Q.E.D.

Again, note that if $\lambda = 1$, so that the research good is produced entirely with labor, then a change in marginal trade costs has no indirect effect on aggregate productivity. Any increase in aggregate productivity that results from changes in firms' exit and export decisions is exactly offset by a decline in product innovation.

The key intuition for this proposition is that, in the absence of productivity dynamics, there are no option values associated with the decisions of exiting and exporting and the marginal firms earn zero current profits from those two activities. Hence, at the margin, changes in the exit and export decisions have no first-order effects on an entering firm's expected profits in the steady state. With $\beta = 1$, the aggregate allocation of labor remains unchanged. All this implies that the ratio of indirect to direct effects on aggregate productivity is the same here as in the version of the model in which all firms export. Hence, as long as the fixed and marginal trade costs are chosen to match the same share of exports in the output of intermediate goods, the response of aggregate productivity in the steady-state to a given percentage change in marginal trade costs is the same, whether all firms export or not.

Note that in Proposition 3, we rely on the assumption that $\beta = 1$ in order to use Lemma 2 to show that L_r is independent of D. This lemma does not apply when $\beta < 1$ and not all firms export. We can extend Proposition 3 to allow for $\beta < 1$. Consider the same version of the model with $\Delta_z = 0$ and a time-invariant level of n_x. Suppose that, in addition, the productivity distribution of entering firms G is such that $\exp(z)$ is Pareto (as in the work of Arkolakis et. al. 2008, Baldwin and Robert-Nicoud 2008 and Chaney 2008). We show in the Appendix that L_r is also unchanged with D, and the ratio of the indirect effect on productivity of a change in marginal trade costs D to the direct effect of this change in trade costs is also given by (4.4).

4.3. Exogenous Selection

Now, we consider the responses of firm process and product innovation and aggregate productivity to a reduction in the costs of international trade in a version of the model with productivity dynamics when not all firms export. We do so in a version of our model in which firms' exit and export decisions are exogenous. Here, a change in marginal trade costs results in a reallocation of process innovation across firms. This reallocation is a portion of the indirect effect of a change in marginal trade costs on productivity that is not present in the two earlier cases, when all firms export or when there are no productivity dynamics. Despite this reallocation of process innovation, we show that (4.4) still applies.

In this version of our model, we assume that the fixed costs of operating n_f equal zero and that the fixed costs of exporting, n_x, follow a two-state Markov process in which $n_x \in \{l, h\}$, with $l = 0$ and $h = \infty$, with a Markov transition matrix

$$\Gamma = \begin{pmatrix} \gamma_l & 1-\gamma_l \\ 1-\gamma_h & \gamma_h \end{pmatrix},$$

with $\gamma_l \geq 1/2$ and $\gamma_h \geq 1/2$. All entering firms start with productivity $z = 0$; and with probability g_i they have $n_x = i$ for $i = l, h$. With these assumptions, firms' exit and export decisions are exogenous and independent of current productivity z. This feature of the equilibrium of this version of our model is what makes it analytically tractable. We refer to our model with these parameters as the *exogenous-selection* version of our model.

Lemma 3: In a symmetric steady-state equilibrium in the exogenous-selection version of our model, the firms' value functions $V(z, n_x)$ have the form $V_i \exp(z)$ for $i = l, h$, and the process innovation decisions $q(z, n_x)$ have the form q_i for $i = l, h$, where V_i and q_i solve

$$V_l = \Pi_d \left(1 + D^{1-\rho}\right) - c(q_l) + \beta(1-\delta)\alpha_l \left[\gamma_l V_l + (1-\gamma_l) V_h\right],$$

$$V_h = \Pi_d - c(q_n) + \beta(1-\delta)\alpha_h \left[\gamma_h V_h + (1-\gamma_h) V_l\right],$$

$$q_i \in \arg\max_{q \in [0,1]} \; -c(q) + \beta(1-\delta)\alpha_i \left[\gamma_i V_i + (1-\gamma_i) V_{-i}\right] \text{ for } i = l, h, \tag{4.6}$$

with α_i denoting the expected growth rate of productivity for continuing firms, given by

$$\alpha_i = \left[q_i \exp(\Delta_z) + (1-q_i)\exp(-\Delta_z)\right].$$

In this symmetric steady state, we have $q_l \geq q_h$.

The value of Π_d is determined by the free-entry condition

$$n_e = \beta\left(g_l V_l + g_h V_h\right), \tag{4.7}$$

and the indices of aggregate productivity Z_d and Z_x solve

$$\begin{pmatrix} Z_x \\ Z_d \end{pmatrix} = (1-\delta) A \begin{pmatrix} Z_x \\ Z_d \end{pmatrix} + \begin{pmatrix} g_l \\ g_h \end{pmatrix}, \text{ with} \tag{4.8}$$

$$A = \begin{pmatrix} \alpha_l \gamma_l & \alpha_h (1-\gamma_h) \\ \alpha_l (1-\gamma_l) & \alpha_h \gamma_h \end{pmatrix}.$$

The aggregates values of W/P, Y, L_r, M_e, Y_r, and C are the solutions to (2.6), (3.8), (3.9), (3.10), (3.11), and (3.12), with

$$\Upsilon = n_e + c(q_l) Z_x + c(q_h) Z_d.$$

Proof: The characterization of the value functions follows because firms never pay fixed costs of operating or exporting, so these fixed costs drop out of the Bellman equation (2.9). It follows immediately that the value functions and process innovation decisions which we put forward solve that Bellman equation. Observe that $V_l > V_h$ because $\gamma_l \geq 1/2$, $\gamma_h \geq 1/2$, and $1 + D^{1-\rho} > 0$. Then, since $c(\cdot)$ is convex, from (4.6) we have that $q_l \geq q_h$, with this inequality strict if $q_i \in (0, 1)$. The intuition for this result is straightforward. Exporters have a bigger market. Because the exporting status is persistent, they also expect to have a bigger market in the future. Hence, they have a greater incentive to innovate.

The aggregate productivity indices of equation (4.8) can be understood as follows. A fraction δ of firms exit exogenously every period. All continuing exporters have an expected productivity growth rate of α_l. A fraction γ_l of these firms remain exporters, and a fraction $(1 - \gamma_l)$ become non-exporters. Likewise, all continuing non-exporters have an expected productivity growth rate of α_h and a transition of export status determined by γ_h. All entering firms have a productivity index $z = 0$ and, hence, productivity of 1. A fraction g_l of these entrants are exporters, and the remainder are non-exporters. Q.E.D.

We now study the impact of a reduction in trade costs on this economy. From the free-entry condition (4.7), we see that a reduction in trade costs must raise the value of exporting firms, V_l, and lower the value of non-exporting firms, V_h. If export status is sufficiently persistent, then the incentives for process innovation, captured in (4.6), increase for exporters and decrease for non-exporters, leading to a reallocation of process innovation across firms.

We can obtain analytical results regarding the impact of the reduction in trade costs on aggregate productivity in this special case of our model if we also set $\beta = 1$.

Proposition 4: In a symmetric steady state in the exogenous-selection version of our model with $\beta = 1$, to a first-order approximation, the ratio of the indirect effect to the direct effect on aggregate productivity of a change in marginal trade costs D is given by (4.4).

Proof: We obtain this result regarding a change in marginal trade costs by differentiating the Bellman equation and the free-entry condition to obtain the steady-state change in profits, and then we obtain the result from (4.2). In particular, differentiating the Bellman equation, with $\beta = 1$, gives that

$$\Delta V_l = \left(1 + D^{1-\rho}\right) \Delta\Pi_d + \Pi_d \Delta \left(1 + D^{1-\rho}\right) + (1 - \delta)\, \alpha_l \left[\gamma_l \Delta V_l + (1 - \gamma_l)\, \Delta V_h\right] \text{ and}$$

$$\Delta V_h = \Delta\Pi_d + (1 - \delta)\, \alpha_h \left[\gamma_h \Delta V_h + (1 - \gamma_h)\, \Delta V_l\right],$$

where we have used an envelope condition to cancel out the terms that arise from marginal changes in process innovation. Writing these in vector form, we obtain that

$$\begin{pmatrix} \Delta V_l \\ \Delta V_h \end{pmatrix} = (1 - (1-\delta) A')^{-1} \begin{pmatrix} (1 + D^{1-\rho}) \Delta \Pi_d + \Pi_d \Delta (1 + D^{1-\rho}) \\ \Delta \Pi_d \end{pmatrix}. \tag{4.9}$$

Free-entry requires that

$$g_l \Delta V_l + g_h \Delta V_h = 0.$$

Together, the last two expressions and the fact that $[1 - (1-\delta) A']^{-1} = \left([1 - (1-\delta) A']^{-1}\right)'$ imply that

$$\begin{aligned} 0 &= \begin{pmatrix} g_l & g_h \end{pmatrix} \left([1 - (1-\delta) A']^{-1}\right)' \begin{pmatrix} (1 + D^{1-\rho}) \Delta \Pi_d + \Pi_d \Delta (1 + D^{1-\rho}) \\ \Delta \Pi_d \end{pmatrix} \\ &= \begin{pmatrix} Z_x & Z_d \end{pmatrix} \begin{pmatrix} (1 + D^{1-\rho}) \Delta \Pi_d + \Pi_d \Delta (1 + D^{1-\rho}) \\ \Delta \Pi_d \end{pmatrix}, \end{aligned} \tag{4.10}$$

where the last equality follows from (4.8). This then implies (4.3). Proposition 4 is obtained by plugging (4.3) into (4.2) and taking into account that L_r is independent of D. Q.E.D.

From Proposition 4, observe that if $\lambda = 1$, then there is no indirect effect of a reduction in trade costs on aggregate productivity in the steady state. Hence, in this case, the impact of the change in process innovation on the productivity of the average firm must be exactly offset by the change in product innovation. More generally, recall that the impact of a change in trade costs on process innovation is independent of the parameter λ. In equilibrium, product innovation is what must adjust differently depending on the parameter λ.

We now discuss how the results of Proposition 4 vary if $\beta < 1$ in the model with exogenous selection. For this analysis, it is useful to define *hybrid* indices of aggregate productivity, $\tilde{Z}_x$ and $\tilde{Z}_d$, as

$$\begin{pmatrix} \tilde{Z}_x \\ \tilde{Z}_d \end{pmatrix} = (1-\delta) \beta A \begin{pmatrix} \tilde{Z}_x \\ \tilde{Z}_d \end{pmatrix} + \begin{pmatrix} g_l \\ g_h \end{pmatrix}. \tag{4.11}$$

Note that in these definitions, we use expression (4.8), where the effective survival rate is $\beta (1-\delta)$ instead of $(1-\delta)$. The hybrid share of exports in intermediate good output, $\tilde{s}_x$, is defined by expression (3.6), with $\tilde{Z}_x$ and $\tilde{Z}_d$ in place of Z_x and Z_x. This hybrid share of exports in intermediate good output corresponds to the share of exports in the discounted present value of revenues for an entering firm. If $\beta = 1$, we have that $\tilde{s}_x = s_x$. If $\beta < 1$, and if entering firms are less (or more) likely to be exporters relative to old surviving firms, then $s_x > \tilde{s}_x$ (or $s_x < \tilde{s}_x$).

Following the same logic as in Proposition 4, we can show that

$$\Delta \log \Pi_d = (1-\rho)\left(\frac{\tilde{s}_x}{s_x}\right) \text{(Direct Effect)}. \tag{4.12}$$

Note that if entering firms are likely to be non-exporters (low g_l), and if export status is persistent, then $\tilde{s}_x$ is close to zero, and aggregate variable profits Π_d are roughly unchanged with D. Then (4.6) implies that process innovation by exporters will increase much more than that by non-exporters. In contrast, if entering firms are likely to be exporters (high g_h), then $\tilde{s}_x$ is high, and Π_d falls by more with D. This larger decline in aggregate variable profits leads to a smaller increase in process innovation by exporting firms than non-exporting firms. Hence, the average export status of entering firms will largely determine the reallocation of process innovation in response to a change in trade costs.

The result (4.12) raises the possibility that the indirect effect on aggregate productivity of a change in trade costs could offset, rather than amplify, the direct effect. In particular, if process innovation is assumed to be highly inelastic, then $\Delta \log Z_x = \Delta \log Z_d = 0$ and then using Lemma 1 and (4.2), we can show that the ratio of the indirect effect to the direct effect is

$$\frac{\text{Indirect Effect}}{\text{Direct Effect}} = -1 + \frac{\rho - 1}{\rho + \lambda - 2}\left[\frac{L_r}{L} + \frac{\tilde{s}_x}{s_x}\left(1 - \frac{L_r}{L}\right)\right]. \tag{4.13}$$

This expression is negative when $\tilde{s}_x/s_x$ is small and λ is large. For example, if $\lambda = 1$, then the indirect effect partly offsets the direct effect if and only if $\tilde{s}_x < s_x$.

4.4. Transition Dynamics

So far, we have focused on steady-state comparisons. We can also compute transitions in our model out of the steady state, although to take into account all the general equilibrium effects, we must do that numerically. In our quantitative analysis in the next section, we find that this model can have very slow transition dynamics despite the fact that the only state variable is the distribution of productivities across firms. We can here gain some intuition for this result in advance, however, by considering equation (4.8) in the exogenous-selection version of the model, interpreted as a first-order difference equation for the aggregate productivity indices Z_{xt} and Z_{dt}.

That equation implies that if q_{lt}, q_{ht}, and M_{et} change once and for all after a one-time change in trade costs in period 0, then the transition dynamics of the aggregate productivity indices are given by

$$\left(\begin{array}{c} Z_{xt}-\bar{Z}_x \\ Z_{dt}-\bar{Z}_d \end{array}\right)=(1-\delta)^t A^t \left(\begin{array}{c} Z_{x0}-\bar{Z}_x \\ Z_{d0}-\bar{Z}_d \end{array}\right), \tag{4.14}$$

where $\bar{Z}_x$ and $\bar{Z}_d$ denote the new steady-state values of these indices. Note that A is a matrix with all non-negative elements and that in order to have a steady state, $(1-\delta)^t A^t$ must converge to zero. If that happens rapidly, then the transition dynamics are fast. If $(1-\delta)^t A^t$ dies out slowly, then the transition dynamics are slow.

This matrix $(1-\delta)A$ also determines in our model the productivity of the average firm relative to that of the average entering firms. On average, entering firms have productivity $[(1+D^{1-\rho}) \;\; 1]\,[g_l \; g_h]'$, and the average firm has productivity $[(1+D^{1-\rho}) \;\; 1]\sum_{t=0}^{\infty}(1-\delta)^t A^t [g_l \; g_h]'$. Hence, if $(1-\delta)^t A^t$ dies out rapidly, then the productivity of the average firm is similar to the average productivity of an entering firm. Here, process innovation is not playing a big role in determining firms' productivities, and transition dynamics are fast. In contrast, if $(1-\delta)^t A^t$ dies out slowly, so that the productivity of the average firm is substantially larger than the average productivity of an entering firm, then process innovation is playing a big role in determining firms' productivities, but the transition dynamics are slow. Our model thus has a trade-off between the importance of process innovation for firms' productivities and the speed of transition to the steady state.

5. Quantitative Analysis

We now present a quantitative version of our model to extend our results from the previous section on the impact of a change in the marginal costs of international trade on aggregate productivity and welfare to specifications of the model that we cannot solve for analytically. In particular, we consider a specification of our model with both endogenous selection in firms' exit and export decisions and potentially elastic process innovation. We also consider the impact of assuming positive interest rates and large changes in marginal trade costs on our results. We parameterize our quantitative model to make it consistent with some salient of U.S. data on firm size dynamics (in terms of both employment and export status) and firm size distribution.

We then conduct four experiments with our parameterized model to consider the impact of various assumptions on our results. In our first experiment, we find, quantitatively, that, with zero interest rates, the response of aggregate productivity to a small change in trade costs, measured as an elasticity, is quite close to what we found analytically above. We then consider the impact of our assumption of zero interest rates in our next two experiments. In our second experiment, we consider a specification of our quantitative model with positive interest rates and inelastic process innovation. This specification of our model extends the model of Melitz (2003) in allowing for both (exogenous) productivity dynamics and positive interest rates. We find that the response of aggregate productivity in this specification of our quantitative model is quite close to our analytical results. In our third experiment, we consider a specification of our quantitative model with positive interest rates and elastic process innovation. Here we find that it is possible to have a larger steady-state response of aggregate productivity than we have found in our previous analytical and quantitative results. In this experiment, however, it remains the case that the reallocation of process innovation toward exporting firms and the adjustment of product innovation have largely offsetting effects on aggregate productivity. In particular, the responses of average productivity and product innovation are both at least an order of magnitude larger than the response of aggregate productivity. We also show in this experiment that this reallocation of innovation has a small impact on welfare because the transition dynamics to the new steady-state are so slow. Finally, in our fourth experiment, we find that when we allow for larger changes in international trade costs, our conclusions from our third experiment are roughly unchanged.

5.1. Calibration

Table 1 lists all of our benchmark parameters. We choose time periods equal to two months so there are six time periods per year. As we reduce the period length, we keep the entry period of new firms at one year. We parameterize the distribution G of productivity draws and the export costs of entrants, so that all firms enter with a common productivity index $z_0 = 0$ and all firms share a level of fixed costs of exporting n_x that is constant throughout the firm's active life.

We assume that the process innovation cost function has the form $c(q) = h \exp(bq)$, so that the curvature of this function is indexed by the parameter b. If this parameter b is high (or low), so that the curvature is high (or low), then process innovation is highly inelastic (or elastic) to changes in the incentives to innovate. We consider alternative values of b ranging from a very large value ($b = 3,000$), in which the process innovation decisions of firms are highly inelastic and, hence, effectively constant, as in the model of Luttmer (2007a), to lower values ($b = 30$ and 10), in which process innovation decisions are elastic, so that the reallocation of process innovation after a trade cost change is quite large.

The remaining parameters of the model are chosen to reproduce a number of salient features of U.S. data on firm dynamics, the firm size distribution, and international trade. The parameters that we must choose are the steady-state real interest rate $1/\beta$, the total number of workers L, the parameters governing the variance of employment growth for surviving firms Δ_z, the exogenous exit rate of firms δ, the marginal trade costs D, the fixed costs of operation n_f and entry n_e, the fixed costs of exporting n_x, and the parameters of the innovation cost function h and b. We also need to choose the elasticity of substitution across intermediate goods in final output ρ and the share of labor in the production of research goods λ. In our model, the distribution of employment across firms in a symmetric steady state depends on the elasticity parameter ρ only through the trade intensity for firms that do export, given by $D^{1-\rho}/(1+D^{1-\rho})$. Much of our calibration procedure is based on employment data, so we choose $D^{1-\rho}$ as a parameter; hence, our steady-state calibration is invariant to the choice of ρ. For similar reasons, our steady-state calibration is also invariant to the choice of λ.

These parameters are set as follows. We consider two values of β: $\beta = 1$, so that the interest rate is zero, and β set so that the steady-state interest rate (annualized) is 5%. We normalize the number of workers $L = 1$. Several parameters shape the law of motion of firm productivity z (Δ_z, δ, n_e, n_f, n_x, $D^{1-\rho}$, h and b). We choose Δ_z so that the standard deviation of the growth rate of employment of large firms in the model is 25% on an annualized basis. This figure is in the range of those for US publicly traded firms, as reported by Davis et al. (2007).[13] We choose the exogenous exit rate δ so that the model's annual employment-weighted exit rate of large firms is 0.55%, which is consistent with that rate for large firms in the U.S. data.[14] Note that in our model, over the course of one year, large firms do not choose to exit endogenously because they have productivity far away from the threshold productivity for exit. Hence δ determines the annual exit rate of these firms directly. We normalize entry costs $n_e = 1$, and we set the fixed costs of operation $n_f = 0.1$.[15]

Corresponding to each value of the elasticity parameter b ($3,000$, 30, and 10), we choose the parameters n_x, $D^{1-\rho}$, and h to match three observations: (1) the fraction of exports in

[13] We abstract from the trend in employment growth rate volatility discussed by Davis et al. (2006) and pick a number that roughly matches the average for the period 1980-2001.

[14] This is the 1997–2002 average employment-based failure rate of U.S. firms larger than 500 employees, computed using the the Statistics of U.S. Businesses, available online at http://www.sba.gov.

[15] The statistics that we report are invariant to proportional changes in all three fixed costs and h.

the gross output of intermediate goods is $s_x = 7.5\%$; (2) the fraction of total production employment accounted for by exporting firms is $s_x\left(1+D^{1-\rho}\right)/D^{1-\rho} = 40\%$[16]; (3) the shape of the right tail of the firm size distribution matches that in the United States. Here, our calibration procedure is similar to that of Luttmer (2007a). Specifically, we represent the right tail of the distribution of employment across firms in the U.S. data with a function that maps the logarithm of the number of employees $\log(l)$ into the logarithm of the fraction of total employment working in firms with this size or larger. This function is known to be close to linear for large firms. In calibrating the model with inelastic process innovation (fixed q for all firms), we set the model parameters so that the model matches the slope coefficient of this function for firms within a certain size range.[17] To be concrete, we target a slope of -0.2 for firms ranging between $1,000$ and $5,000$ employees.

Note that firm sizes in terms of number of employees in the model are simply a normalization. We choose this normalization so that the median firm in the employment-based size distribution is of size 500. In other words, 50% of total employment in the model is accounted for by firms with fewer than 500 employees.[18] The calibrated model then implies a value of process innovation q for large firms. As we lower b, we adjust the model parameters to keep the value of q for large firms constant and thus keep the dynamics of large firms unchanged.

Table 1 summarizes the numbers used in the calibration, as well as the resulting parameter values, for each level of b. Recall that by calibrating the model to data on firm size, we do not need to take a stand on the values of ρ and λ. The aggregate implications of changes in trade costs are, however, affected by those values. In our benchmark parameterization, we set $\rho = 5$ and λ equal to either 1 or 0.5.[19]

[16] Bernard et al. (2008) report that the fraction of total US empoyment (excluding a few sectors such as agriculture, education, and public services) accounted for by exporters is 36.3% in 1993 and 39.4% in 2000. The average of exports and imports to gross output for the comparable set of sectors is roughly 7.5% in the U.S. in 2000. The steady state of our model abstracts from trends in trade costs that would lead to changes in trade volumes over time.

[17] The slope coefficient for sufficiently large firms can be solved for analytically in our model. In particular, given the choice of process innovation q for large firms, the slope coefficient is $1+\log(y)/\Delta_z$, where y is the root of $y = (1-\delta)\,q + (1-\delta)(1-q)\,y^2$, which is less than 1 in absolute value.

[18] This is, the size of the median firm in the U.S. firm employment-based size distribution on average in the period 1999–2003, as reported by the Statistics of U.S. Businesses, available online at http://www.sba.gov.

[19] Our choice of $\rho = 5$ roughly coincides with the average elasticity of substitution for U.S. imports of differentiated four-digit goods estimated by Broda and Weinstein (2006) for the period 1990–2001.

5.2. Experiment 1: Interest Rate Zero, Process Innovation Elasticity Varying

In our first experiment, we consider the calibration of our model in which the interest rate is 0% and the elasticity of process innovation varies. This calibration of our model combines the endogenous selection of firms' exit and export decisions of the Melitz (2003) model with the productivity dynamics driven by endogenous process innovation. Since the interest rare is zero, we know from Lemma 1 that the aggregate allocation of labor does not change. In this experiment, we evaluate the accuracy of (4.3) and (4.4) derived in our analytical results of Section 3. Here, as well as in Experiments 2 and 3, we reduce marginal trade costs by a small magnitude ($\Delta \log D = -0.005$) and compute the change in the symmetric steady state of the model. We report all changes as elasticities (ratios of changes in the log of the variables to $\Delta \log D$) with a minus sign so that these elasticities can be interpreted as the increase in aggregate productivity, output, and so on in response to a decline in trade costs. We repeat these experiments for our three values of b ($b = 3,000$, 30, and 10), and our two values of λ ($\lambda = 1$ and 0.5) for a total of six parameter configurations of the model. Results are reported in Table 2.

Since the share of exports in intermediate good output is $s_x = 0.075$ and $\rho = 5$, it is clear that for all six of these cases, our analytical formula (4.3) is very accurate. When $\lambda = 1$ [in columns (1)–(3) in Table 2], our formula (4.4) for the ratio of the indirect effect to the direct effect is also quite accurate. In this case, the indirect effect is roughly zero because product innovation adjusts to offset the changes in exit, export, and process innovation. This implies that the aggregate changes in aggregate productivity in these three cases are close to changes from the direct effect alone.

Note that when process innovation is elastic, there is a large reallocation of process innovation from non-exporters to exporters. This reallocation leads to a large change in the share of exports in output. In particular, the elasticity of the export share s_x to a change in D is 3.7 with $b = 3,000$, 9.9 with $b = 30$, and 25.9 with $b = 10$. (We do not report these numbers in Table 2) This reallocation leads to a large increase in the productivity of the average firm (its elasticity is roughly 0 with $b = 3,000$, 1.17 with $b = 30$, and 3.87 with $b = 10$). However, in each case, a large offsetting movement in product innovation leaves the indirect effect of a reduction in marginal trade costs on aggregate productivity roughly unchanged.

For those cases in Table 2 with $\lambda = 0.5$ [columns (4)-(6)] the conclusions are similar, in that the numerical results are close to the analytical predictions. Here the change in aggregate productivity is larger (0.086 instead of 0.075) because the indirect effect is larger, as predicted by (4.4).

From Lemma 2, we have that when the interest rate is zero, a change in trade costs does not affect the steady-state change ratio of consumption to output. This result is confirmed in Table 2: the response of aggregate consumption in this experiment is the same as that of aggregate output.

5.3. Experiment 2: Interest Rate Positive, Process Innovation Inelastic

In our second experiment, we consider the parameterization of our model in which the annualized interest rate is 5% and process innovation is inelastic ($b = 3,000$). This version of the model is the one discussed at the end of the analytic section, extended with endogenous selection in exit and export decisions. We perform the same aggregate exercises here as in Experiment 1, using values of λ equal to 1 and 0.5, and report the results in Table 3.

We find that with these changes, the formulas for the change in the constant in variable profits, (4.12), and the ratio of the indirect to the indirect effects, (4.13), are very accurate. Our main finding from this experiment is that, with a small value of $\tilde{s}_x$, the indirect effect of a reduction in marginal trade costs is negative. That is, the decline in product innovation more than offsets the changes in the productivity of the average firm. Hence, the resulting change in aggregate productivity is *smaller* than those that arise from the direct effect alone. In particular, the direct effect on aggregate productivity is roughly 0.075, which is larger than the resulting change in aggregate productivity reported in columns (1)–(2) in Table 3 (0.03 with $\lambda = 1$ and 0.019 with $\lambda = 0.5$).

This result that the indirect effect is negative is largely driven by the result that the change in variable profits is so small, as given by $(1 - \rho)\,\tilde{s}_x$. The intuition for this result is that entering firms start small, and they take many periods to become exporters. Hence, with a positive interest rate, changes in marginal trade costs do not have a significant impact on the variable profits of entering firms.

To illustrate the importance of firm dynamics for this result, consider an alternative parameterization of our model in which the constant h in the process innovation cost function is set to a higher level, so that entering firms on average do not grow substantially. In this alternative parameterization, s_x and $\tilde{s}_x$ are both roughly equal to 0.075. This parameterization might be relevant for capturing productivity dynamics at the product level rather than at the firm level if we think that new products enter at a relatively larger scale. In this parameterization, entering products are roughly the same size as the average firm and, hence, have a relatively high probability of being exported shortly after entry. When we repeat Experiment 2 with this alternative parameterization of our model, we obtain the results reported in columns (3) and (4) of Table 3. Compared to the first parameterization results, here the change in variable profits is larger in absolute terms, and the indirect effect is roughly zero or slightly positive. In terms of the impact on aggregate productivity, these results are similar to those we obtained in columns (1) and (4) of Table 2 with a zero interest rate. This result suggests that, quantitatively, the hybrid export share $\tilde{s}_x$ plays a large role in determining the effects of a change in marginal trade costs on aggregate productivity in the steady state.

5.4. Experiment 3: Interest Rate Positive, Process Innovation Elastic

In our third experiment, we consider a specification of our model that is not close to one we solved analytically. Exit and export decisions are endogenous, the annualized interest rate is 5% and the values of b governing the elasticity of process innovation are equal to 30 and 10. We report the results in Table 4.

We see in columns (1)–(4) that shifting to this parameterization produces a large reallocation of labor (for example, the elasticity of aggregate production labor with $b = 10$ and $\lambda = 1$ in column (2) is 0.29) and less of an offset of product innovation to the change in the productivity of the average firm (the ratio of the indirect effect to the direct effect on productivity in column (2) is 0.26). From (3.14), we see that both of these effects can contribute to a substantial amplification of the direct effect of a reduction in trade costs on output. The response of aggregate output is also large compared to that seen in columns (1) and (2) of Table 3, which assumes inelastic process innovation. In particular, if $\lambda = 1$, then the elasticity of aggregate output to a reduction in D is 0.03 with $b = 3,000$, 0.15 with $b = 30$, and 0.38 with $b = 10$. Thus, with $b = 10$, the response of output is more than five times what would arise from the direct effect alone. Note, however, that there is still a substantial offsetting effect between process and product innovation. The elasticity of the indirect effect on the productivity of the average firm and the elasticity of product innovation are both at least an order of magnitude larger than their combined effect on aggregate productivity. For example, with $b = 10$ and $\lambda = 1$ in column (2) of Table 4, the elasticity of the productivity of the average firm is 2.64, while that of aggregate productivity is only 0.095.

5.5. Welfare in Experiments 2–3

Our results so far concern the impact of a small change in marginal trade cost on steady-state levels of aggregate productivity and output. Now, we ask whether considering firms' decisions to exit, export, and innovate substantially affect the model's implications for the effects of a change in trade costs on welfare.

Our welfare metric is the equivalent variation in consumption from a change in marginal trade costs, defined as the change in consumption at the old steady state that leaves households indifferent between the old steady state and the transition to the new steady state. To ensure that our welfare measure is well defined, we consider welfare only in those specifications of our model with positive interest rates ($\beta < 1$).

To put these welfare gains in perspective, we compare them to the magnitude of the welfare gains from the same change in trade costs in a specification of our model with only product innovation. In particular, we use as a benchmark a specification of our model in which exit decisions are exogenous, all firms export, and product innovation is inelastic so that there are no indirect effects of a change in marginal trade costs on welfare arising from changes in firms' decisions on these margins. Therefore, by comparing the welfare gains in our calibrated model to the welfare gains found in this benchmark specification of our model, we can determine the importance of changes in firms' exit, export, and product innovation decisions for welfare. We calibrate this benchmark specification of our model to obtain the same baseline share of exports in the output of intermediate goods.

Consider now the welfare implications of a small change in the marginal trade costs in our model as specified in Experiments 2 and 3. In Tables 3 and 4, we report the elasticity of the equivalent variation in consumption with respect to $\Delta \log D$ for both our calibrated model and our benchmark specification.

In both Experiments 2 and 3, as reported in columns (1)–(4) of Tables 3 and 4, we see that our welfare statistic is very similar in both specifications of our model. Hence, in these experiments almost no effects on welfare arise from the indirect effects associated with changes in firms' exit, export, and product innovation decisions and the reallocation of aggregate labor in the transition to a new steady state, despite the fact that both of these sources contribute to a large change in aggregate output and consumption.

These results follow from the fact that when the steady-state response of aggregate productivity and output to a change in marginal trade costs is large, the transition dynamics are very slow and, hence, contribute little to welfare. To illustrate these slow transition dynamics, we plot in Figure 1 the elasticity of the ratio of exports to output of intermediate good firms during the transition. As is evident in the figure, when entering firms are small relative to the average firm, these transition dynamics take more than 100 years to play out. This is consistent with our analytical argument that our model's aggregate transition dynamics are connected to its firm dynamics. When entering firms are small relative to the average firm, aggregate transition dynamics are slow. When entering firms are larger, these dynamics are much faster. To illustrate this point, we also show in Figure 1 the transition dynamics for exports relative to output of intermediate good firms for the specifications of our model in which entering firms are large relative to the average firm, as described in columns (5) and (6) of Table 4. In particular, in these specifications, entering firms on average do not grow substantially, so the actual and hybrid shares of employment in exporters are similar. We see that, for this specification of our model, the aggregate transition dynamics are substantially faster.

Note, however, that despite the faster transition dynamics, our welfare statistics are still roughly the same across specifications of our model because, in the long run, the indirect effect and the aggregate reallocation of labor both contribute to only a small change in aggregate output and consumption.[20]

5.6. Experiment 4: Larger Trade Cost Change

Now we repeat Experiments 2 and 3, but with a larger change in international trade costs. In particular, using the same parameter values as in the earlier experiments, we compute the welfare effects that arise from a 4.5% reduction in D rather than from a very small change.[21]

We report the results in Table 5. Depending on the elasticity of process innovation, this

[20] The result that consideration of firm's exit, export, process innovation decisions have a very small impact on the welfare implications of a change in marginal trade costs can also be understood through the lens of the planning solution of our model. As discussed above, the equilibrium allocations of our model coincide with the planning solution under $\lambda = 1$, or if $\lambda < 1$ in the presence of a per-unit subsidy that eliminates distortionary monopoly markups. In the planning problem, with firms' exit, export, and process innovation decisions optimally chosen, the envelope condition implies that, to a first-order approximation, the increase in the discounted flow of utility from a change in marginal trade costs is equal to the discounted present value of the direct effect of this change on aggregate productivity. We know from the envelope condition that changes in firms' exit, export, and process innovation decisions are of higher than first order for welfare.

[21] We choose this change in trade cost to ensure that our model still has a stationary steady state. Choosing a larger reduction in D with $b = 10$ leads to an even larger increase in the growth rate of exporting firms and a non-stationary firm size distribution. We also computed the welfare gains using a larger change in trade cost with $b = 30$ (i.e., a 15% reduction in D), and found roughly similar results.

large trade cost change results, in the long run, in an increase in the export share from 7.5% to 9.3% when $b = 3,000$, or to 20.5% when $b = 10$. Despite the large change in trade patterns which comes from a reallocation of process innovation from non-exporters to exporters, there is a large offsetting response of product innovation. As reported in Table 5, the change in the productivity of the average firm is at least one order of magnitude larger than the change in aggregate productivity. Overall, the welfare gains that arise from the indirect effects associated with changes in firms' exit, export, and process innovation decisions and the reallocation of aggregate labor in the transition to a new steady state are not very different to our benchmark specification in which all firms export and process innovation is inelastic. For example, with $\lambda = 1$, the welfare gains are 8.15% when all firms export and process innovation is inelastic, and 9.2% when not all firms export and process innovation is elastic with parameter $b = 10$.

6. Concluding remarks

In this paper we have built a model of endogenous change in aggregate productivity that arises in general equilibrium as firms' exit, export, process and product innovation decisions respond to a change in international trade costs. Our central finding is that, despite the fact that such a trade cost change can have a substantial impact on individual firms' decisions, that impact is not reflected in aggregate productivity. In particular, the steady-state response of product innovation largely offsets the impact of changes in firms' exit, export, and process innovation decisions on aggregate productivity. In our quantitative exercise, we also find that the dynamic welfare gains from a reduction in trade costs are not substantially larger than those from simpler models that abstract from endogenous selection and process innovation, despite the fact that changes in firms' exit, export and process innovation decisions lead to very large dynamic responses of exports and the firm size distribution. Our results thus suggest that microeconomic evidence on individual firms' exit, export and process innovation responses to changes in international trade costs is not likely to be informative about the macroeconomic implications of these change for aggregate productivity and welfare.

Our model has abstracted from four important considerations. First, we have assumed a continuum of firms with constant elasticity of demand, which implies that changes in trade costs have no impact on firms' markups and that process innovation decisions do not strategically interact across firms. Our model could be extended to allow for variable markups. (For a model of trade and heterogeneous firms with non-constant elasticity of demand, see the work of Melitz and Ottaviano (2008). For models of process innovation with strategic interactions between firms, see those of Ericson and Pakes (1995) and Aghion et al. (2001).)

Second, we have assumed that all firms produce only one good. In doing so, we have ignored the effects that a change in trade costs might have on product innovation by incumbent firms. Consideration of process and product innovation in models with multi-product firms would be an important extension of our work here. (For different types of models of multi-product firms, see the work of Klette and Kortum (2004), Luttmer (2007b), and Bernard, Redding and Schott (2009).)

Third, we have largely abstracted from asymmetries across countries. Depending on the assumed form of the asymmetries, terms of trade effects not present in the symmetric case can arise. These terms of trade effects are associated with a reallocation of process and product innovation across countries.

Finally, we have abstracted from spillover effects that might lead to endogenous growth. Given the work of Baldwin and Robert-Nicoud (2008) on the role of spillovers in the Melitz model, we expect that our model could generate a wide variety of results depending on the details of the spillovers.

Bloom, Draca, and Van Reenen (2009) present evidence that innovation, measured in a variety of ways, rises in import competing industries relative to other industries as the volume of trade rises. Extensions of our model along our proposed directions may be useful in accounting for this observation.

One other extension of our work could be fruitful. Here we have focused on how firms' decisions to innovate are affected by changes in international trade costs. More generally, a broader array of other economic changes and policies could affect these decisions. We conjecture that our main result here regarding the role of the free-entry condition in constraining the response of aggregate productivity would extend to the effect of other types of changes as well, for example, various policies designed to stimulate firm innovation. For these and other changes, the response of process innovation by existing firms might be offset in equilibrium by a change in product innovation. We leave consideration of this sort of extension for future work.

References

[1] Aghion, Philippe, Christopher Harris, Peter Howitt and John Vickers (2001) "Competition, Imitation, and Growth with Step-by-Step Innovation," Review of Economic Studies 68(3): 467–492.

[2] Arkolakis, Costas (2008) "Market Penetration Costs and Trade Dynamics," Manuscript, Yale University, New Haven, CT.

[3] Arkolakis, Costas, Svetlana Demidova, Peter Klenow, and Andrés Rodriguez-Clare (2008) "Endogenous Variety and the Gains from Trade," American Economic Review 98(2): 444–450.

[4] Aw, Bee Yan, Mark J. Roberts, and Daniel Yi Xu (2009). "R&D Investment, Exporting, and Productivity Dynamics," NBER Working Paper 14670. Cambridge, MA.

[5] Bajona, Claustre, Mark J. Gibson, Timothy J. Kehoe, and Kim J. Ruhl (2008) "Trade Liberalization, Growth, and Productivity," Manuscript, University of Texas, Austin.

[6] Baldwin, Richard E. , and Frédéric Robert-Nicoud (2008) "Trade and Growth with Heterogenous Firms," Journal of International Economics, 74(1): 21–34.

[7] Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel S. Kortum (2003) "Plants and Productivity in International Trade," American Economic Review 93(4): 1268–1290.

[8] Bernard, Andrew, J. Bradford Jensen, and Peter Schott (2008) "Importers, Exporters, and Multinationals: A Portrait of Firms in the U.S. that Trade Goods" in T. Dunne, J. B. Jensen and M. J. Roberts (eds.), Producer Dynamics: New Evidence from Micro Data, University of Chicago Press, Chicago, IL.

[9] Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott (2007) "Firms in International Trade," Journal of Economic Perspectives 21(3):105–130.

[10] Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott (2009) "Multi-Product Firms and Trade Liberalization," Manuscript, Dartmouth College, Hanover, NH.

[11] Bloom, Nicholas, Mirko Draca, and John Van Reenen (2009) "Trade Induced Technical Change? The Impact of Chinese Imports on Innovation and Information Technology," Manuscript. Stanford University, Palo Alto, CA.

[12] Broda, Christian and David E. Weinstein (2006) "Globalization and the Gains from Variety," Quarterly Journal of Economics, 121(2): 541–585.

[13] Bustos, Paula (2007) "The Impact of Trade on Technology and Skill Upgrading: Evidence from Argentina," Manuscript, Universitat Pompeu Fabra, Barcelona.

[14] Chaney, Thomas (2008) "Distorted Gravity: The Intensive and Extensive Margins of International Trade," American Economic Review, 98(4): 1707-1721.

[15] Costantini, James and Marc J. Melitz (2008) "The Dynamics of Firm Level Adjustment to Trade Liberalization," in E. Helpman, D. Marin, and T. Verdier (eds.), The Organization of Firms in a Global Economy, pp. 107-141, Harvard University Press, Cambridge, MA.

[16] Davis, Steven J., John C. Haltiwanger, Ron Jarmin, and Javier Miranda (2007) "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms," in D. Acemoglu, K. Rogoff, and M. Woodford (eds.), NBER Macroeconomics Annual 2006, 21(1): 107–156, MIT Press, Cambridge, MA.

[17] De Loecker, Jan (2007) "Do Exports Generate Higher Productivity? Evidence from Slovenia," Journal of International Economics, 73(1): 69–98.

[18] Doraszelski, Ulrich, and Jordi Jaumandreu (2008) "R&D and Productivity: Estimating Production Functions when Productivity is Endogenous" Manuscript, Harvard University, Cambridge, MA.

[19] Eaton, Jonathan and Samuel Kortum (2001) "Technology, Trade, and Growth: A Unified Framework," European Economic Review: Papers and Proceedings, 45(4–6), 742–755.

[20] Ericson, Richard, and Ariel Pakes (1995) "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," Review of Economic Studies 62(1): 53–82.

[21] Griliches, Zvi (1979) "Issues in Assessing the Contribution of Research and Development to Productivity Growth," Bell Journal of Economics, 10(1): 92-116.

[22] Grossman, Gene M. and Elhanan Helpman (1991) Innovation and Growth in the Global Economy. MIT Press, Cambridge, MA.

[23] Hopenhayn, Hugo (1992) "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," Econometrica 60(5): 1127-1150.

[24] Helpman, Elhanan (2006) "Trade, FDI, and the Organization of Firms" Journal of Economic Literature 44(3): 589-630.

[25] Irarrazabal, Alfonso A. and Luca David Opromolla (2008) "A Theory of Entry and Exit into Export Markets," Working Paper 2008-20. Banco de Portugal, Lisboa.

[26] Klette, Tor Jakob, and Samuel Kortum (2004) "Innovating Firms and Aggregate Innovation," Journal of Political Economy, 112(5): 986-1018.

[27] Krugman, Paul R. (1979) "Increasing Returns, Monopolistic Competition, and International Trade" Journal of International Economics, 9(4): 469–479.

[28] Lentz, Rasmus, and Dale T. Mortensen (2008) "An Empirical Model of Growth Through Product Innovation," Econometrica, 76(6): 1317–1373.

[29] Lileeva, Alla, and Daniel Trefler (2007) "Improved Access to Foreign Markets Raises Plant-Level Productivity for Some Plants," NBER WP 13297. National Bureau of Economic Research, Cambridge, MA.

[30] Long, Ngo Van, Horst Raff, and Frank Stähler (2008) "Innovation and Trade with Heterogeneous Firms," Kiel Working Paper 1430, Kiel Institute for the World Economy, Kiel, Germany.

[31] Luttmer, Erzo G.J. (2007a) "Selection, Growth, and the Size Distribution of Firms," Quarterly Journal of Economics, 122(3): 1103–1144.

[32] Luttmer, Erzo G.J. (2007b) "New Goods and the Size Distribution of Firms," Research Department Working Paper 649, Federal Reserve Bank of Minneapolis, Minneapolis, MN.

[33] Melitz, Marc J. (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," Econometrica, 71(6): 1695-1725.

[34] Melitz, Marc J., and Ottaviano Gianmarco I.P. (2008) "Market Size, Trade, and Productivity," Review of Economic Studies, 75(1): 295–316.

[35] Navaz-Ruiz, Antonio, and Davide Sala (2007) "Technology Adoption and the Selection Effect of Trade," Economics Working Paper EC02007/58, European University Institute.

[36] Rivera-Batiz, Luis A., and Paul M. Romer (1991) "Economic Integration and Endogenous Growth," Quarterly Journal of Economics, 106(2): 531–555.

[37] Yeaple, Stephen R. (2005) "A Simple Model of Firm Heterogeneity, International Trade, and Wages," Journal of International Economics, 65(1): 1–20.

7. Appendix: Derivations and Proofs

Aggregate Variables in the Symmetric Steady State

Here we derive the equations defining the aggregate variables in the symmetric steady state. The definition of the price index of the final good in the home country (2.4) implies that the real wage is given by (3.8). Using (3.5), the labor market clearing condition (2.14) can be expressed as

$$L = \left(\frac{\rho - 1}{\rho}\right)^{\rho} \left(\frac{W}{P}\right)^{-\rho} Y M_e \left[Z_d + \left(1 + D^{1-\rho}\right) Z_x\right] + L_r. \tag{7.1}$$

From (3.8) and (7.1), aggregate output is given by (3.9).

From (2.6) and (3.8), the resource constraint on the research good, (2.15), can be expressed as

$$\Upsilon M_e = \left(\frac{1-\lambda}{\lambda}\frac{\rho-1}{\rho}\right)^{1-\lambda} \left(M_e \left[Z_d + \left(1 + D^{1-\rho}\right) Z_x\right]\right)^{\frac{1-\lambda}{\rho-1}} L_r. \tag{7.2}$$

From (2.7), the constant on variable profits (3.1) in a symmetric steady state is given by (3.11). From (3.8) and (3.9), the constant on variable profits can be written as

$$\Pi_d = \frac{\lambda^{\lambda} (1-\lambda)^{1-\lambda}}{\rho^{1-\lambda} (\rho - 1)^{\lambda}} \left[M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)\right]^{\frac{2-\rho-\lambda}{\rho-1}} (L - L_r). \tag{7.3}$$

Pre-multiplying (7.3) by $M_e \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right)$, dividing this expression by (7.2), and rearranging terms, we obtain (3.10), the employment used to produce the research good.

We also know, from (2.13), that $C = Y - Y_r$, or from (2.6), that $C = Y - L_r\left[\left(1-\lambda\right)/\lambda\right]\left(W/P\right)$. Using (3.8), (3.9), and (3.10), we obtain (3.12). Note that $0 \leq C \leq Y$, because $\zeta \geq 1$ and $\rho > 1$. Q.E.D.

Lemma 2

Here we prove Lemma 2, which is introduced in Section 3. The last two lines in equation (2.16) define an operator that maps existing distributions of firms across states into new distributions of firms across states. We denote this operator by T, and re-write (2.16) as

$$M_{t+1} = TM_t + GM_{et}.$$

Hence, the steady-state distribution of firms across states, scaled by the measure of entering firms, is given by

$$\tilde{M} = \sum_{n=0}^{\infty} T^n G.$$

This distribution is the sum of the firm distribution across those that are from $n = 0$ to $n = \infty$ periods old.

Note that if we integrate our Bellman equation (2.9) with $\beta = 1$, then with respect to any arbitrary distribution of firms across states $H(s)$, we get

$$\int V(s)\ \mathrm{d}H(s) =$$

$$\int \left[\Pi_d\left(1 + x(s)\,D^{1-\rho}\right)\exp(z) - x(s)\,n_x - n_f - c\left(q(s)\exp(z)\right)\right]\ \mathrm{d}H(s) + \int V(s)\ \mathrm{d}TH(s).$$

Iterating on this expression, using G as the initial distribution in place of H, gives that

$$\int V(s)\ \mathrm{d}G(s) = \sum_{n=0}^{\infty} \int \left[\Pi_d\left(1 + x(s)\,D^{1-\rho}\right)\exp(z) - x(s)\,n_x - n_f - c\left(q(s)\exp(z)\right)\right] \mathrm{d}T^n G.$$

Using the free-entry condition (2.11) then gives

$$n_e = \sum_{n=0}^{\infty} \int \left[\Pi_d\left(1 + x(s)\,D^{1-\rho}\right)\exp(z) - x(s)\,n_x - n_f - c\left(q(s)\exp(z)\right)\right] \mathrm{d}T^n G.$$

Reversing the order of summation and integration gives Lemma 2. Q.E.D.

Proposition 3

Here we provide additional details for Proposition 3 in the version of our model with $\Delta_z = 0$ and time-invariant fixed export costs n_x so that there are no dynamics in productivity or export decisions. To simplify our presentation, we assume that there is a single (as well as time-invariant) value of n_x, but our results carry through if there are multiple levels of n_x. We allow for $\beta < 1$.

The steady-state value of a firm with productivity z is given by

$$V(z) = \frac{1}{1-\beta(1-\delta)} \max\left\{0, \Pi_d \exp(z) - n_f + \max\left\{0, \Pi_d \exp(z) D^{1-\rho} - n_x\right\}\right\}. \tag{7.4}$$

The free-entry condition is

$$\beta \int V(z)\ \mathrm{d}G(z) = n_e\ ,$$

where $G(z)$ is the distribution of the productivity of entering firms.

The exit cutoff $\bar{z}$ is defined by $\Pi_d \exp(\bar{z}) = n_f$, and the export cutoff $\bar{z}_x$ by $\Pi_d D^{1-\rho} \exp(\bar{z}_x) = n_x$. We assume, without loss of generality, that $n_f < n_x D^{\rho-1}$, so that the export cutoff is strictly higher than the exit cutoff.

Using the value functions (7.4), we can write the free-entry condition as

$$\Pi_d \left[\delta Z_d + \left(1 + D^{1-\rho}\right) \delta Z_x\right] - \left[1 - G(\bar{z})\right] n_f - \left[1 - G(\bar{z}_x)\right] n_x = \frac{\left[1 - \beta(1-\delta)\right]}{\beta} n_e\ , \tag{7.5}$$

where the indices of aggregate productivity scaled by the measure of entering firms are

$$Z_d = \frac{1}{\delta} \int_{\bar{z}}^{\bar{z}_x} \exp(z)\ \mathrm{d}G(z) \ \text{ and } Z_x = \frac{1}{\delta} \int_{\bar{z}_x}^{\infty} \exp(z)\ \mathrm{d}G(z).$$

Differentiating (7.5), we obtain

$$\Delta\Pi_d \delta \left[Z_d + \left(1 + D^{1-\rho}\right) Z_x\right] + \Pi_d \delta Z_x \Delta \left(1 + D^{1-\rho}\right)$$

$$+ \left[n_f - \Pi_d \exp(\bar{z})\right]\ \mathrm{d}G(\bar{z})\, \Delta\bar{z} + \left(n_x - \Pi_d D^{1-\rho}\right) \exp(\bar{z}_x)\ \mathrm{d}G(\bar{z}_x)\, \Delta\bar{z}_x = 0.$$

Using the cutoff definitions, we can drop out the last two terms, so that

$$\Delta\Pi_d \left[Z_d + \left(1 + D^{1-\rho}\right) Z_x\right] + \Pi_d Z_x \Delta \left(1 + D^{1-\rho}\right) = 0,$$

which results in (4.3).

The average expenditures on the research good per entering firm, Υ, from (3.7) are

$$\Upsilon = n_e + \frac{1 - G(\bar{z})}{\delta} n_f + \frac{1 - G(\bar{z}_x)}{\delta} n_x. \tag{7.6}$$

The free-entry condition (7.5) can be expressed, with the use of (7.6), as

$$\Pi_d \left(Z_d + \left(1 + D^{1-\rho}\right) Z_x\right) - \Upsilon = \frac{(1-\beta)}{\delta\beta} n_e. \tag{7.7}$$

If $\beta = 1$, then $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon = 1$ (which confirms Lemma 1). With Lemma 2, that expression implies that L_r is unchanged with D. Hence, the ratio of the indirect effect to the direct effect of changes in trade costs on aggregate productivity is given by (4.4).

We now show that, if we allow for $\beta < 1$ and assume that G is such that $\exp(z)$ is distributed Pareto, we get that $\zeta = \Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] / \Upsilon$ is unchanged with D. Thus, Lemma 1 applies, and hence, L_r is unchanged with D. Therefore, the ratio of the indirect effect to the direct effect of changes in trade costs on aggregate productivity is again given by (4.4).

In particular, we assume that the cumulative distribution function of $\exp(z)$ is

$$G(\exp(z)) = 1 - \left(\frac{\exp(z_0)}{\exp(z)}\right)^{\sigma}, \text{ for } \exp(z) > \exp(\bar{z}_0).$$

Under this assumption, we have that

$$Z_d = \int_{\exp(\bar{z})}^{\exp(\bar{z}_x)} \frac{\sigma}{\delta} \frac{\exp(\bar{z}_0)^{\sigma}}{\exp(z)^{\sigma}} \, \mathrm{d}\exp(z) = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} \left[\exp(\bar{z})^{1-\sigma} - \exp(\bar{z}_x)^{1-\sigma}\right] \text{ and}$$

$$Z_x = \int_{\exp(\bar{z}_x)}^{\infty} \frac{\sigma}{\delta} \frac{\exp(\bar{z}_0)^{\sigma}}{\exp(z)^{\sigma}} \mathrm{d}\exp(z) = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} \exp(\bar{z}_x)^{1-\sigma}.$$

Using the definitions, we get that

$$Z_d = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} (\Pi_d)^{\sigma-1} \left[n_f^{1-\sigma} - \left(\frac{n_x}{D^{1-\rho}}\right)^{1-\sigma}\right], \text{ and}$$

$$Z_x = \frac{\sigma \exp(\bar{z}_0)^{\sigma}}{\delta(\sigma - 1)} (\Pi_d)^{\sigma-1} \left(\frac{n_x}{D^{1-\rho}}\right)^{1-\sigma}.$$

Therefore, we have

$$\Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] = \frac{\sigma}{\delta(\sigma - 1)} \left[\exp(\bar{z}_0) \Pi_d\right]^{\sigma} \left[n_f^{1-\sigma} + n_x^{1-\sigma} \left(D^{1-\rho}\right)^{\sigma}\right]. \tag{7.8}$$

Using the cutoff definitions, we can express Υ as

$$\Upsilon = n_e + \frac{1}{\delta} \left[\exp(\bar{z}_0) \Pi_d\right]^{\sigma} \left[n_f^{1-\sigma} + n_x^{1-\sigma} \left(D^{1-\rho}\right)^{\sigma}\right]. \tag{7.9}$$

Combining (7.8) and (7.9), we obtain

$$\Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right] = \frac{\sigma}{\sigma - 1} (\Upsilon - n_e)$$

Combined with (7.7), this implies that both $\Pi_d \left[Z_d + Z_x \left(1 + D^{1-\rho}\right)\right]$ and Υ are independent of D. Therefore, Lemma 1 applies.

In this case, if $\lambda = 1$, M_e is invariant to changes in D. Hence, it is possible to prove this result without the use of the free-entry condition, but instead fixing the number of firms in each country. One does require the free-entry condition to prove our result when $\lambda < 1$.

Figure 1 : Transition Dynamics of Exports/Output from a Decline in Marginal Trade Costs

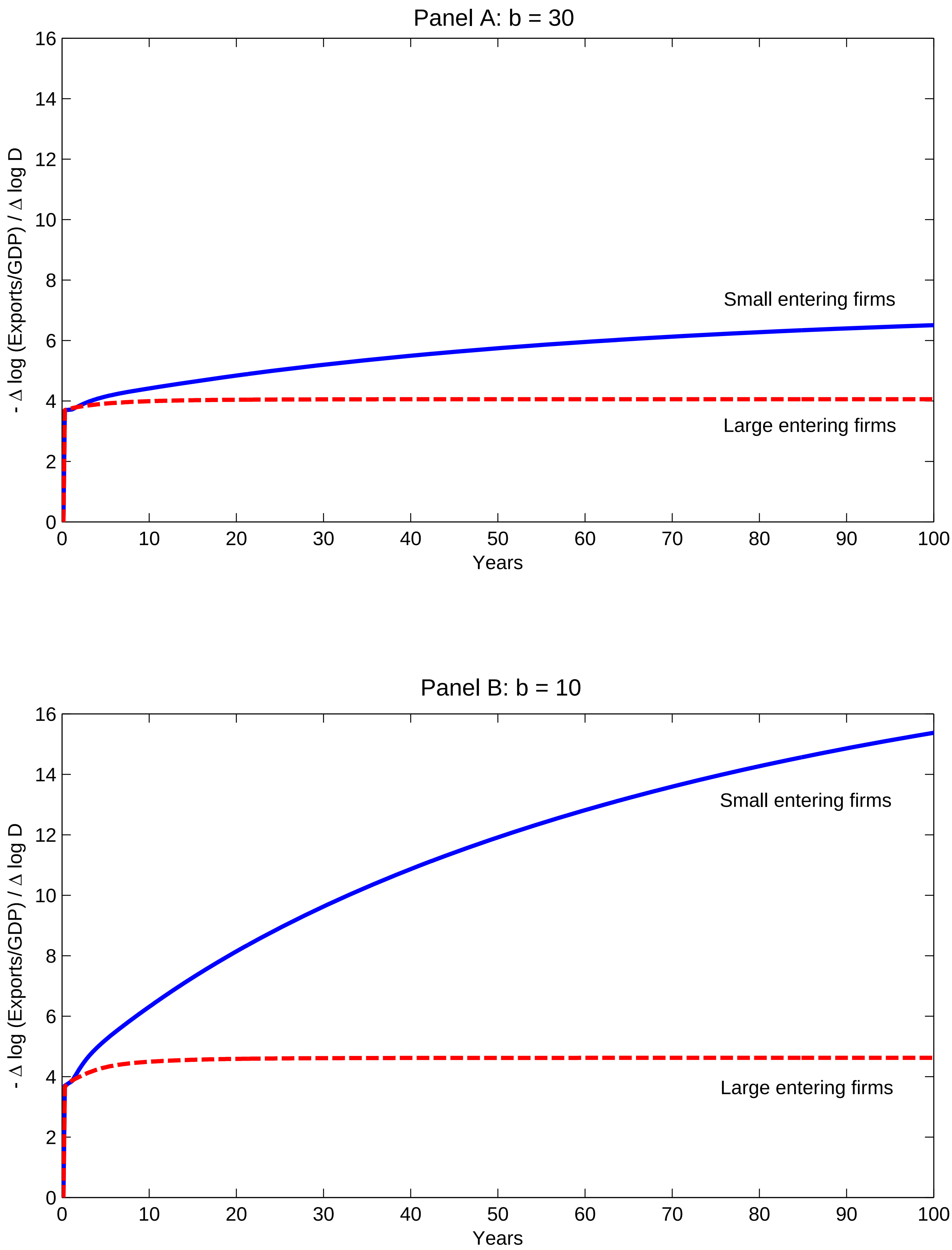

TABLE 1: Model Parameterization

		(1)	(2)	(3)
		Curvature of process innovation cost function, *b*		
		3,000	30	10
Calibrated Parameters				
Exogenous exit rate, δ (annualized)		0.005	0.005	0.005
Process innovation step size, Δ_z (annualized)		0.25	0.25	0.25
Level of process innovation cost function, *h* (or employment-based right-tail coefficient of large firms)		1.3E+42 (-0.25)	0.31 (-0.25)	0.49 (-0.25)
Marginal trade costs , $D^{(1-\rho)}$		0.231	0.231	0.231
Fixed costs of international trade, n_x		1.4	0.7	0.285
Targets	**U.S. Data**			
Employment growth rate of large firms, (annual standard deviation)	0.25	0.25	0.25	0.25
Annual employment-based exit rate, firms with more than 500 employees	0.0055	0.0055	0.0055	0.0055
Employment-based right tail coefficient, firms from 1,000 to 5,000 employees	-0.2	-0.21		
Exports / GDP (of intermediate goods in model)	0.075	0.076	0.076	0.075
Employment share of exporters (production employment in model)	0.40	0.41	0.41	0.40

Other Parameters

Annualized interest rate, $1/\beta$ annualized = 0 and 0.05
Share of labor in production of research good, λ = 1 and 0.5
Elasticity of substitution across intermediate goods, ρ = 5
Fixed entry cost, n_e = 1
Fixed operation cost, n_f = 0.1

TABLE 2

Experiment 1: Effects of a Small Reduction in Marginal Trade Costs, With Zero Interest Rate

	(1)	(2)	(3)	(4)	(5)	(6)
	Research good produced with labor only (λ=1)			Research good produced with labor + goods (λ=0.5)		
Parameters						
Curvature of process innovation cost function, *b*	3,000	30	10	3,000	30	10
Export share, s_x	0.075	0.075	0.076	0.075	0.075	0.076
Hybryd export share, $\tilde{s}_x$	0.075	0.075	0.076	0.075	0.075	0.076
Elasticity of aggregate variables across steady states						
negative of (log change in variable / log change in *D*)						
Constant on variable profits, Π_d	-0.302	-0.300	-0.303	-0.302	-0.300	-0.303
Aggregate productivity, *Z*	0.075	0.075	0.076	0.086	0.086	0.087
Direct effect	0.075	0.075	0.076	0.075	0.075	0.076
Productivity of the average firm	0.000	1.168	3.850	0.000	1.168	3.850
Product Innovation	0.000	-1.170	-3.872	0.011	-1.160	-3.862
Aggregate Production Labor, $L-L_r$	0.000	0.000	0.000	0.000	0.000	0.000
Output, *Y*	0.075	0.076	0.076	0.086	0.086	0.087
Consumption, *C*	0.075	0.076	0.076	0.086	0.086	0.087
Ratio of indirect / direct effects, Numerical	0.00	0.00	0.00	0.14	0.14	0.14
Ratio of indirect / direct effects, Theoretical	0.00	0.00	0.00	0.14	0.14	0.14

TABLE 3

Experiment 2: Effects of a Small Reduction in Marginal Trade Costs, With Positive Interest Rate and Inelastic Process Innovation

	(1)	(2)	(3)	(4)
	Small entering firms		Large entering firms	
	λ =1	λ =0.5	λ =1	λ =0.5
Parameters				
Curvature of process innovation cost function, *b*	3,000	3,000	3,000	3,000
Export share, sx	0.076	0.076	0.078	0.078
Hybryd export share, $\tilde{s}_x$	0.004	0.004	0.075	0.075
Elasticity of aggregate variables across steady states				
negative of (log change in variable / log change in *D*)				
Constant on variable profits, Π_d	-0.017	-0.017	-0.301	-0.301
Aggregate productivity, *Z*	0.009	0.008	0.076	0.086
Direct effect	0.076	0.076	0.078	0.078
Productivity of the average firm	0.000	0.000	0.000	0.000
Product Innovation	-0.067	-0.068	-0.002	0.008
Aggregate Production Labor, $L\text{-}L_r$	0.020	0.011	0.002	0.001
Output, *Y*	0.030	0.019	0.077	0.087
Consumption, *C*	0.030	0.028	0.077	0.088
Ratio of indirect / direct effects, Numerical	-0.88	-0.89	-0.03	0.11
Ratio of indirect / direct effects, Theoretical	-0.88	-0.90	-0.03	0.11
Welfare	0.076	0.073	0.078	0.087
Welfare in benchmark (all firms export, exog. exit)	0.075	0.077	0.075	0.077

TABLE 4

Experiment 3: Effects of a Small Reduction in Marginal Trade Costs, With Positive Interest Rate and Elastic Process Innovation

	(1)	(2)	(3)	(4)	(5)	(6)
	Small entering firms				Large entering firms	
	λ =1		λ =0.5		λ =1	
Parameters						
Curvature of process innovation cost function, *b*	30	10	30	10	30	10
Export share, sx	0.076	0.075	0.076	0.075	0.075	0.077
Hybryd export share, $\tilde{s}_x$	0.009	0.022	0.009	0.022	0.073	0.072
Elasticity of aggregate variables across steady states negative of (log change in variable / log change in *D*)						
Constant on variable profits, Π_d	-0.035	-0.090	-0.035	-0.090	-0.291	-0.289
Aggregate productivity, *Z*	0.037	0.095	0.027	0.071	0.074	0.076
Direct effect	0.076	0.075	0.076	0.075	0.075	0.077
Productivity of the average firm	0.626	2.660	0.626	2.660	0.046	0.258
Product Innovation	-0.666	-2.651	-0.675	-2.675	-0.047	-0.259
Aggregate Production Labor, $L\text{-}L_r$	0.112	0.291	0.060	0.159	0.005	0.015
Output, *Y*	0.148	0.387	0.087	0.230	0.079	0.091
Consumption, *C*	0.148	0.387	0.142	0.383	0.079	0.091
Ratio of indirect / direct effects, Numerical	-0.52	0.26	-0.64	-0.06	-0.02	-0.01
Welfare	0.076	0.076	0.076	0.078	0.075	0.077
Welfare in benchmark (all firms export, exog. exit)	0.075	0.075	0.077	0.073	0.075	0.075

TABLE 5

Experiment 4: Effects of a Large Reduction in Marginal Trade Costs, With Positive Interest Rate and Elastic Process Innovation

	(1)	(2)	(3)	(4)	(5)	(6)
	Research good produced with labor only (λ=1)			Research good produced with labor + goods (λ=0.5)		
Parameters						
Curvature of process innovation cost function, *b*	3,000	30	10	3,000	30	10
Export share, Initial steady state	0.076	0.076	0.075	0.076	0.076	0.075
Export share, New steady state	0.093	0.110	0.206	0.093	0.110	0.206
Elasticity of aggregate variables across steady states						
negative of (log change in variable / log change in *D*)						
Constant on variable profits, Π_d	-0.020	-0.043	-0.122	-0.020	-0.043	-0.122
Aggregate productivity, *Z*	0.007	0.042	0.195	0.007	0.032	0.137
Direct effect + Productivity of the average firm (*)	0.109	0.923	14.488	0.109	0.923	14.488
Product Innovation	-0.102	-0.881	-14.293	-0.102	-0.892	-14.351
Aggregate Production Labor, $L\text{-}L_r$	0.006	0.127	0.659	0.003	0.068	0.358
Output, *Y*	0.013	0.169	0.854	0.010	0.099	0.495
Consumption, *C*	0.013	0.169	0.854	0.012	0.161	0.833
Welfare	0.084	0.086	0.092	0.082	0.086	0.096
Welfare in benchmark (all firms export, exog. exit)	0.081	0.081	0.081	0.084	0.084	0.084

(*): We do not separately report the direct and indirect effects on average productivity because equation (4.1) is not very precise with a large change in *D* .