The Advantage of Flexible Targeting Rules*

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Abstract

This paper investigates the consequences of debt stabilization for inflation targeting. If the monetary authority perfectly stabilizes inflation while the fiscal authority holds constant the real value of debt at maturity, the equilibrium dynamics might be indeterminate. However, determinacy can be restored by committing to targeting rules for either monetary or fiscal policy that include a concern for stabilization of the output gap. In solving the indeterminacy problem, flexible inflation targeting appears to be more robust than flexible debt targeting to alternative parameter configurations and steady-state fiscal stances. Conversely, flexible fiscal targeting rules lead to more desirable welfare outcomes. The paper further shows that if considerations beyond stabilization call for a combination of strict inflation and debt targeting rules, the indeterminacy result can be overturned if the fiscal authority commits to holding constant debt net of interest rate spending.

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1 Introduction

Inflation targeting is becoming more and more the dominant paradigm in monetary policy-making. Following the pioneering experience of New Zealand in 1990, as of today more than 20 countries, both in the industrialized and developing world, have decided to adopt this approach to monetary policy conduct (Svensson, 2007).

Although less popular, fiscal targeting rules have also been introduced in practice as a device to impose accountability on governments' discretionary decisions. Most notably, balanced budget rules, adopted for instance in several U.S. states, Canadian provinces and in the provisions of the Maastricht Treaty for EMU partners (the Stability and Growth Pact), can in fact be interpreted as debt targeting rules.[1]

This paper studies the determinacy properties of the equilibrium in an economy in which both fiscal and monetary authorities set policy according to targeting rules. The starting point for the analysis is that the combination of a strict inflation targeting rule for monetary policy with a strict debt targeting rule for fiscal policy generates equilibrium indeterminacy for a wide range of parameter configurations and steady state fiscal stances.

In spite of this negative result, targeting rules remain attractive for a variety of reasons.[2] The accountability criterion, mentioned above in relation to fiscal policy, is certainly important for monetary authorities too in order to build reputation. In general, targeting rules simplify the task of communication and make quite straightforward for the private sector to verify whether a certain criterion has been met by the policymaker.

These benefits motivate the search for alternative formulations of the rules that could allow for solving the indeterminacy problem while remaining within the realm of a targeting framework. The rest of the paper shows that targeting rules that include a concern for stabilization of the output gap have the potential to meet those two requirements. Yet, the analysis also highlights the existence of a tradeoff. While flexible inflation targeting rules (of the type advocated by Svensson, 2003) appear to be robust in terms of determinacy to a wider range of parameter configurations, flexible debt targeting rules are associated with substantially more desirable welfare properties. This tradeoff can be resolved by committing to a flexible fiscal targeting rule that stabilize debt net of interest rate payments. This rule avoids the spiral feedback of monetary onto fiscal policy which is at the heart of the indeterminacy result and at the same time still admits a flexible formulation with desirable

[1] In this paper, the word "targeting" does not necessarily reflect variables entering the policymakers' loss function, as in Svensson (2003). Rather, it refers to setting policy as to meet a certain "target" for some macro variable(s).

[2] See, for instance, Bernanke et al. (1999) on the benefits of inflation targeting

welfare properties.

The analysis is developed in the context of the economy discussed in Benigno and Woodford (2003). This model retains much of the tractability of the baseline New Keynesian framework (Clarida et al., 1999, and Woodford, 2003), with the additional feature, crucial for the purpose of this paper, of fiscal policy playing an active stabilization role.[3]

The existing literature has typically studied the determinacy properties of the equilibrium under targeting rules for either fiscal policy (as in Schmitt-Grohé and Uribe, 1997) or monetary policy (as in Giannoni and Woodford, 2002, and Svensson and Woodford, 2003).[4]

The existing contribution closest to this work is the paper by Benhabib and Eusepi (2005), who examine the conditions for local and global determinacy in a model in which the monetary authority follows an interest rate rule and the fiscal authority either aims at balancing the budget or sets taxes in response to the level of real debt at maturity.[5] In that paper, indeterminacy arises very much for the same reasons discussed here. The authors show that interest rate rules that respond to fluctuations in the output gap help to solve the indeterminacy problem.

This paper finds that such a result extends to the case of inflation targeting. Moreover, an additional contribution of this work is to demonstrate that the indeterminacy outcome can alternatively be overturned by allowing the fiscal authority to respond to fluctuations in the output gap. While this finding appears to be less robust than for flexible inflation targeting, flexible debt targeting rules display superior welfare properties. The alternative specification of debt targeting rules discussed in the last section of this paper adds yet another possible solution to the indeterminacy problem. By targeting debt net of interest payments, this formulation of fiscal rules turns out to be robust to alternative parametrizations while still achieving higher welfare than flexible inflation targeting alone.

The next section presents the model. Section 3 shows that the equilibrium is indeterminate in case of strict inflation and debt targeting rules. Section 4 demonstrates that a commitment to flexible policy rules for both monetary and fiscal policy restores determinacy of the equilibrium and investigates the robustness of the result to alternative parameter configurations and steady state fiscal stances. Section 5 derives the optimal policy benchmark and computes the optimal weights on the output gap for the flexible targeting rules, together

[3] The model in fact nests the baseline New Keynesian model as a special case in which lump-sum taxes satisfy the government budget constraint in each period.

[4] A large body of the literature has examined the determinacy properties of the equilbrium under feedback rules. For instance, Leeper (1991) presents an early discussion of interest rate and tax rate feedback rules in a flexible price model. Taylor (1999) (as well as several other contributions in the same volume) specifically examine the performance of interest rate rules in monetary models with nominal rigidities.

[5] See also Schmitt-Grohé and Uribe (2006) for an application to a medium scale DSGE model.

with the consumption equivalent associated with each policy. Finally, section 6 introduces the alternative debt targeting rule that avoids indeterminacy even in case of a strict formulation. The last section concludes.

2 The Model

This section presents the log-linear approximation of the model discussed in Benigno and Woodford (2003).[6] The appendix reports the details which are summarized here for expositional convenience. The economy is populated by a continuum of households of measure one. Preferences over consumption and leisure are time-additive and separable. Households receive compensation for their labor supply and earn financial income from dividends and the realization of a portfolio of state-contingent securities. An exogenous wage markup shock acts as a wedge between the real wage and the marginal rate of substitution between consumption and leisure. The consumption index is a CES aggregator of differentiated varieties. Firms purchase labor inputs from households taking wages as given and produce according to a decreasing return to scale technology, with an economy-wide productivity shock. Prices are set on a staggered basis. The fiscal authority (government) levies distortionary sales taxes and issues one period nominal debt to finance a given stream of wasteful spending and transfer shocks. The monetary authority (central bank) decides upon the nominal interest rate. Money is the unit of account but it does not circulate explicitly as an asset in the economy (cashless limit).

The model can be summarized by an aggregate supply relation (Phillips curve), the government budget constraint and an aggregate demand relation (Euler equation).

The Phillips curve has the standard forward looking form but it is augmented by a term which represents the "tax gap"

$$\pi_t = \kappa \left[y_t + \psi \left(\hat{\tau}_t - \hat{\tau}_t^*\right)\right] + \beta E_t \pi_{t+1}. \tag{1}$$

In expression [1], π_t is the inflation rate, y_t is the welfare-relevant output gap and $\hat{\tau}_t - \hat{\tau}_t^*$ is the tax gap.[7] The tax rate $\hat{\tau}_t$ enters the Phillips curve because taxes directly affect the firms'

[6] As discussed in Ferrero (2008), the linearized model presented in this paper in fact encompasses other formulations of fiscal policy than the one discussed in Benigno and Woodford (2003). The exact quantitative results, however, crucially depend on the specific formulation adopted.

[7] The targets for output and taxes are linear combinations of the exogenous shocks. In general, these targets do not correspond to either the efficient or the natural levels. For the purpose of this paper, the exact form of the targets is in fact irrelevant. See Benigno and Woodford (2003) for the exact definitions and a discussion of several special cases.

pricing decisions. The term $\hat{\tau}_t^*$ is the level of the tax rate that guarantees contemporaneous stabilization of inflation and the output gap in the absence of fiscal policy considerations.

The government budget constraint can be written in flow form as

$$\hat{B}_t = (1-\beta)\left[b_y y_t + b_\tau \left(\hat{\tau}_t - \hat{\tau}_t^*\right)\right] + \beta E_t \hat{B}_{t+1}, \tag{2}$$

where

$$\hat{B}_t \equiv \hat{b}_{t-1} - \pi_t - \sigma^{-1} y_t + f_t. \tag{3}$$

In expression [3], $\hat{b}_t$ is the real value of government debt at maturity and f_t is the "fiscal stress".[8] Compared to the baseline New Keynesian model, the presence of an additional policy instrument (the tax rate) implies that cost-push shocks can be completely offset by closing the tax gap. The fiscal stress summarizes the exogenous disturbances which impede the achievement of contemporaneous inflation and output gap stabilization. On the other hand, if lump-sum taxes were available, the government solvency condition [2] would cease to be a constraint on the optimal policy problem and the entire tax gap could simply be treated as the exogenous cost-push shock. The model would then coincide with the baseline version of the New Keynesian framework.

The Phillips curve and the government budget constraint suffice to describe the equilibrium to the extent that the specification of fiscal and monetary policy includes no reference to the nominal interest rate, which would then be defined residually by the Euler equation

$$r_t = r_t^* + E_t \pi_{t+1} + \sigma^{-1}\left(E_t y_{t+1} - y_t\right), \tag{4}$$

where r_t^* is the interest rate that would prevail in case of complete stabilization of inflation and the output gap. More generally, the Euler equation [4] completes the description of the equilibrium for any couple of fiscal and monetary rules that close the model.

3 Strict Targeting Rules and Indeterminacy

This section shows that, under reasonable calibrations, strict targeting rules for fiscal and monetary policy lead to an indeterminate equilibrium. Table 1 reports the original calibration in Benigno and Woodford (2003) which serves as a benchmark for the numerical check of the

[8] In Benigno and Woodford (2003), the fiscals stress is a linear combination of productivity, wage markup, government spending and fiscal transfer shocks. For simplicity, this paper assumes that the fiscal stress follows a first order autoregressive process with persistence ρ_f. This assumption is totally innocuous for the determinacy results, which constitute the main point of the paper. Nor it alters the relative welfare rankings presented below, although it is likely to influence their absolute magnitudes.

Table 1: Baseline calibration and implied steady state values.

β	$=$	0.99	DISCOUNT FACTOR
σ^{-1}	$=$	0.157	ADJUSTED COEFFICIENT OF RISK AVERSION
ω	$=$	0.473	ADJUSTED INVERSE FRISCH ELASTICITY
κ	$=$	0.0236	SLOPE OF THE PHILLIPS CURVE
θ	$=$	10	ELASTICITY OF SUBSTITUTION AMONG VARIETIES
Φ	$=$	1/3	S.S. OVERALL DISTORTION
$\bar{\tau}$	$=$	20%	S.S. TAX RATE
$\bar{b}/\left(4\bar{Y}\right)$	$=$	60%	S.S. ANNUALIZED DEBT-GDP RATIO

determinacy conditions.

The monetary authority is assumed to follow an inflation targeting rule that allows for deviations from full price stability according to the rate of change of the output gap

$$\pi_t + \gamma\left(y_t - y_{t-1}\right) = 0, \tag{5}$$

where the coefficient γ controls the intensity of the feedback from real activity onto inflation.

The fiscal authority is assumed to follow a debt targeting rule that stabilizes debt with some concern for the level of the output gap

$$\hat{b}_t + \lambda y_t = 0, \tag{6}$$

where the coefficient λ represents the sensitivity of the response of debt to real activity.

In general, the concept of flexibility is open to a variety of interpretations. Rules [5] and [6] specifically introduce a concern for the welfare-relevant output gap as an example of departure from a strict formulation of the policy rules. The emphasis here is on the comparison between strict and flexible specification of targeting rules, while at the same time keeping the model analytically tractable.

Rule [5] constitutes a natural benchmark for the analysis of monetary policy in the context of inflation targeting. In inflation targeting countries, central banks typically combine the objective of price stability with a concern for some measure of real activity (see Svensson, 2008). Moreover, in the baseline New Keynesian model, expression [5] (provided that γ is an appropriate function of the structural parameters) represents in fact the optimal monetary policy under commitment and brings about a determinate rational expectation equilibrium.[9]

[9] See Woodford (2003). Evans and Honkapohja (2006) discuss the implementability issues associated with

In that model, however, determinacy would be ensured also by a strict inflation targeting rule that achieves complete price stability in every period ($\gamma = 0$).

Expression [6] also represents a sensible starting point for the analysis of fiscal policy under targeting rules. The rule is formulated here as a restriction on the path of government debt but can easily be recast in terms of budget requirement by substituting [6] into the government budget constraint [2]. The strict version of the rule ($\lambda = 0$) corresponds to the special case of balanced budget in each period, a constraint on fiscal authorities often used, or at least discussed, in practice, as the experience of several U.S. states and the Stability and Growth Pact in the European Monetary Union suggest.

The first result of this paper that motivates much of the following analysis is that if both policy authorities follow a strict formulation of their targeting rules, the resulting equilibrium is indeterminate.

Suppose the monetary and fiscal authorities commit to follow rules [5] and [6] with $\gamma = 0$ and $\lambda = 0$ respectively. From the Phillips curve [1], it is then possible to express the tax gap as a function of the output gap and to substitute the result into [2]. The outcome is a forward looking first order difference equation in the output gap

$$\beta\sigma^{-1}E_t y_{t+1} = \left[(1-\beta)\left(b_y - \psi^{-1}b_\tau\right) + \sigma^{-1}\right] y_t - u_{f,t}, \tag{7}$$

where $u_{f,t} \equiv f_t - \beta E_t f_{t+1}$. The solution of [7] delivers a determinate equilibrium if and only if

$$\left|\rho_y\right| > 1, \qquad \text{where } \rho_y \equiv \frac{(1-\beta)\left(b_y - \psi^{-1}b_\tau\right) + \sigma^{-1}}{\beta\sigma^{-1}}. \tag{8}$$

Under the baseline calibration, the inequality in [8] is not satisfied and the equilibrium under strict inflation targeting and constant debt rules is indeterminate. Figure 1 plots the determinacy regions as a function of the steady state tax rate, holding fixed all the other parameters. Interestingly, indeterminacy occurs for the range of steady state tax rates in which the majority of tax revenues as a fraction of GDP observed in industrialized economies actually falls. Values of the steady state tax rate below 20% are of little interest because, holding constant the ratio between government spending and GDP, the implied steady state government debt would be negative. On the other extreme, values above 40%, while possibly more relevant in practice, imply an economy lying on the "slippery" slope of the Laffer curve (see Trabandt and Uhlig, 2006). In particular, the value of the steady state tax rate that

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the optimal targeting rules [5] when lump-sum taxes are available. The analysis in this paper is confined to equilibrium determinacy.

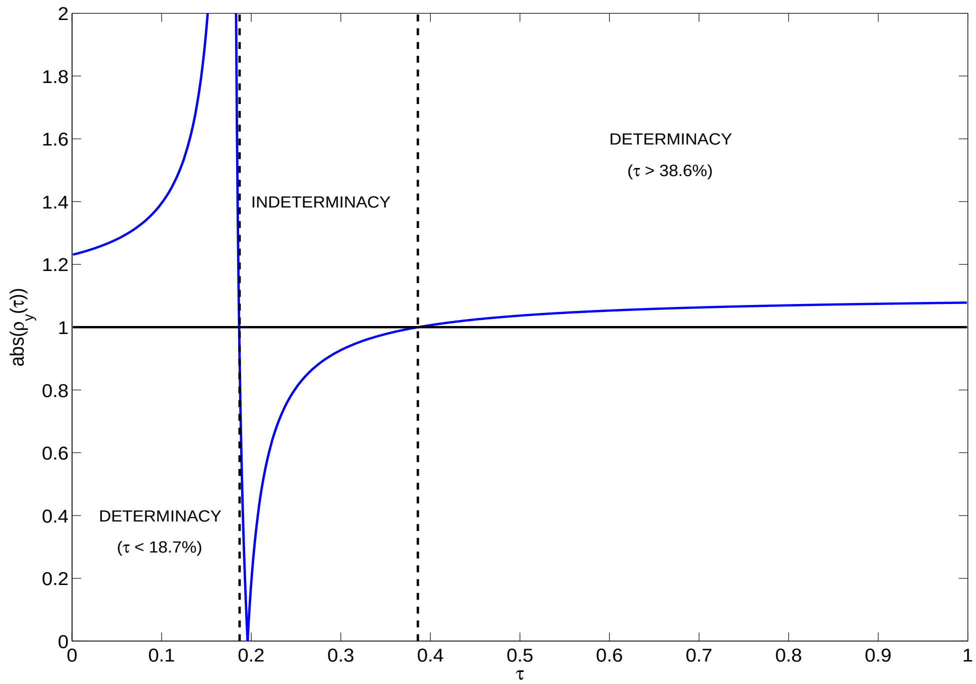

Figure 1: Absolute value of ρ_y as a function of $\bar{\tau}$.

maximizes government revenues is

$$\bar{\tau}^* = 1 - \frac{1}{1 + \left(\omega + \tilde{\sigma}^{-1}\right)} = 39.89\%,$$

while the upper threshold for determinacy is

$$\bar{\tau}^h = 1 - \frac{1}{1 + \left(\omega + \sigma^{-1}\right)} = 38.65\%. \tag{9}$$

The difference between the two tax rates lies in the correction for the steady state consumption share which accounts for the differential percentage point.

The indeterminacy result under strict targeting rules is very much robust to alternative calibrations. The right hand side of [9] is increasing in the sum of the adjusted inverse Frisch elasticity of labor supply ω and the adjusted coefficient of relative risk aversion. This sum is equal to 0.63 under the baseline calibration. If the production function is linear in labor, the parameter ω is actually equal to the inverse Frisch elasticity of labor supply. Holding fixed the steady state output share of consumption to the baseline calibration (equal to 82.4%), the actual coefficient of risk aversion is approximately equal to 0.13. Hence, the values for ω and σ^{-1} adopted by Benigno and Woodford (2003) are clearly at the lower end of the spectrum of what macroeconomists generally use. For instance, Galí et al. (2007) choose a benchmark of 5 for the coefficient of relative risk aversion and 1 for the inverse Frisch elasticity but also consider values of 1 for the former and 2 and 5 for the latter. All combinations would imply a value of $\bar{\tau}^h$ of at least 67% to ensure a determinate equilibrium, hence widening the indeterminacy region.

Schmitt-Grohé and Uribe (1997) show that balanced budget rules generate indeterminacy in the neoclassical growth model for reasonable values of the steady state tax rate. In their paper, however, the indeterminacy result crucially depends on the presence of capital. With no capital, the equilibrium is determinate. Here, indeterminacy occurs also in the absence of capital. The reason is that the strict fiscal policy rule tries to stabilize the real value of debt at maturity and hence creates a feedback effect on monetary policy decisions via the nominal interest rate. Intuitively, an exogenous shock with inflationary consequences leads the monetary authority to hike the nominal interest rate in order to stabilize inflation. The higher interest rate increases the value of debt at maturity and hence puts pressure on the current fiscal budget. To balance the budget, the government raises the tax rates. But this action generates inflationary pressures that forces the monetary authority to increase the interest rate even further. If the feedback is too strong, as in the case of strict rules, the outcome is an unstable path for the output gap even in presence of stationary processes for the exogenous shocks.

The bottom line is that strict targeting rules for monetary and fiscal policy can endanger the stability of the economy by bringing about indeterminate equilibria. This possibility is very much robust to alternative parametrizations. In fact, it is not unconceivable to imagine a situation in which the indeterminacy region depicted in figure 1 actually widens.

On the other hand, targeting rules remain appealing for a variety of other reasons, such as, for instance, the simplicity of communication and the accountability of the policymakers' performances. The point of this paper is not to dismiss the usefulness of targeting rules based on the indeterminacy results of this section. The objective is rather to emphasize the concept of flexibility in the design of the appropriate targeting rules from a variety of perspectives, starting with determinacy but also discussing stabilization outcomes and welfare properties. The remaining sections undertake these questions.

4 Escaping Indeterminacy via Flexible Targeting Rules

This section shows that a commitment to a flexible targeting rule, either by the monetary or by the fiscal authority, is often sufficient to avoid an indeterminate equilibrium.

4.1 Flexible Inflation Targeting

This section demonstrates that an inflation targeting rule like [5] with $\gamma > 0$ generally ensures a determinate equilibrium when the fiscal authority follows a strict debt rule of the form $\hat{b}_t = 0$. In other words, a flexible inflation targeting rule alone is sufficient to avoid the indeterminacy result of the previous section even if fiscal policy remains constrained by a balanced budget requirement.

As in the previous section, a solution for the tax gap can be obtained from the Phillips curve. The result can then be substituted into the government budget constraint together with the fiscal and monetary rules to eliminate debt and inflation. The resulting expression is a second order difference equation in the output gap

$$\beta \left(\omega_\gamma - \sigma^{-1}\right) E_t y_{t+1} - \left[\beta \left(\omega_\gamma - \sigma^{-1}\right) + \omega_\gamma + \beta\sigma^{-1} \left(1 - \rho_y\right)\right] y_t + \omega_\gamma y_{t-1} = u_{f,t}, \qquad (10)$$

where $\omega_\gamma \equiv \gamma \left[1 + (1-\beta)(\kappa\psi)^{-1} b_\tau\right]$. It is straightforward to check that, for $\gamma = 0$, expression [10] coincides with [7] in the previous section.

The determinacy properties of the model in case of flexible inflation targeting and strict debt rules depend upon the roots of the characteristic equation

$$\mathcal{P}(\mu) \equiv \mu^2 - \left[1 + \frac{\omega_\gamma + \beta\sigma^{-1}\left(1-\rho_y\right)}{\beta\left(\omega_\gamma - \sigma^{-1}\right)}\right] \mu + \frac{\omega_\gamma}{\beta\left(\omega_\gamma - \sigma^{-1}\right)} = 0.$$

Equation [10] is equivalent to a system of two equations in two unknowns with one predetermined and one forward looking variable. Hence, the equilibrium is determinate if and only if the absolute value of the roots of $\mathcal{P}(\mu)$ lie on opposite sides of the unit circle. A necessary and sufficient condition for this to be true is that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$ or vice versa.[10]

Under the baseline calibration, the equilibrium is determinate for any value of γ larger than 0.0047. A very small amount of flexibility to monetary policy is sufficient to move the economy out of the indeterminacy region discussed in the previous section. The intuition is that endowing the monetary authority with the flexibility to respond to some measure of real

[10] See the appendix for a formal proof.

activity limits the adverse spillover of fiscal policy onto monetary policy. Consider again an exogenous shocks with inflationary consequences. First of all, in this case, the hike in the interest rate will be less steep than in the case of strict inflation targeting, hence generating a smaller negative feedback on the government budget. Moreover, while the government will still need to increase taxes given the balanced budget prescription, the inflationary consequence of this action will be partly mitigated by the monetary authority concern for real activity. Therefore, the further raise in the interest rate induced by fiscal policy will be smaller. The quantitative analysis suggests that the degree of flexibility necessary to avoid indeterminacy is rather small.

The threshold for the flexibility parameter γ that ensures determinacy is increasing in the adjusted coefficient of risk aversion but decreasing in the adjusted inverse Frisch elasticity of labor supply. Hence, alternative combinations of those two parameters in line with the literature, such as those discussed in the previous section, are likely to preserve the result that a small amount of flexibility to monetary policy brings about a determinate equilibrium. For instance, if both σ^{-1} and ω are equal to 1, the threshold for determinacy is $\gamma_{\min} = 0.012$. If $\sigma^{-1} = 5$ and $\omega = 2$, the threshold moves up slightly to $\gamma_{\min} = 0.0192$.

Section 3 also suggests that the robustness analysis for alternative values of the steady state tax rate is relevant in the interval $[18.7\%, 38.6\%]$, which includes the ratio between tax revenues and GDP for most industrialized countries. The threshold $\gamma_{\min}$ is not very sensitive to variations of the tax rate $\bar{\tau}$ in the interval of interest. The threshold is increasing in the steady state tax rate but, at the peak of the Laffer curve, the amount of flexibility in the inflation targeting rule necessary to ensure determinacy is still rather small: $\gamma_{\min}(\bar{\tau} = 38.6\%) = 0.0439$. Figure 2 helps to visualize these results.

4.2 Flexible Debt Targeting

This section proves that also a commitment to a flexible debt targeting rule like [6] ensures a determinate equilibrium for a wide set of $\lambda > 0$, when the monetary authority follows a strict inflation targeting rule of the form $\pi_t = 0$. In other words, it is possible to design a flexible debt targeting rule that prevents the indeterminacy result of section 3 even if monetary policy continues to fully stabilize inflation in every period. The main difference with the previous section is the more limited spectrum of values for λ that guarantee determinacy. Therefore, the design of flexible debt rules calls for particular attention to the choice of the actual degree of feedback of output onto debt variability.

Once again, a solution for the tax gap can be obtained from the Phillips curve and then substituted into the government budget constraint together with the fiscal and monetary

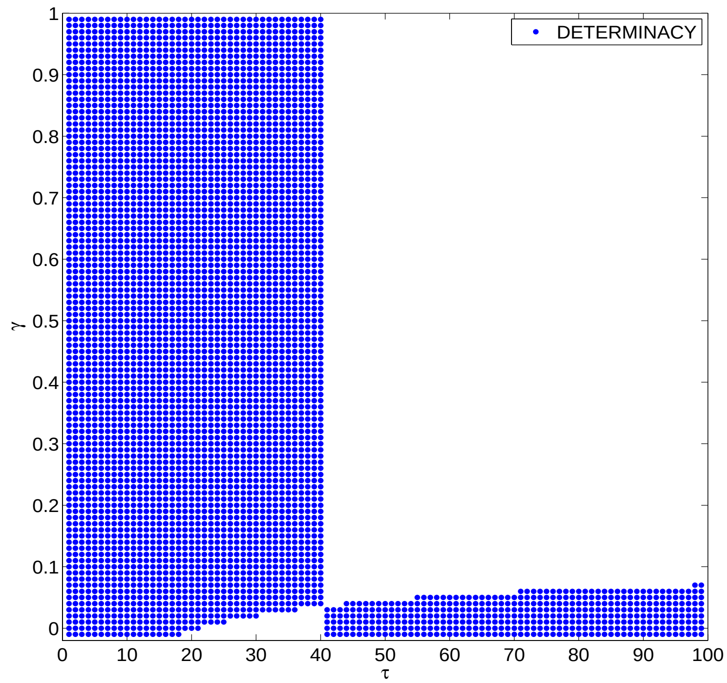

Figure 2: Flexible inflation targeting and determinacy.

rules to eliminate debt and inflation. The resulting expression is

$$\beta\sigma^{-1}E_t y_{t+1} - \beta\left(\sigma^{-1}\rho_y - \lambda\right)y_t - \lambda y_{t-1} = -u_{f,t}. \tag{11}$$

Obviously, if $\lambda = 0$, expression [11] and [7] coincide. If $\lambda > 0$, the determinacy properties of the model depend on the roots of the characteristic equation

$$\mathcal{P}(\mu) \equiv \mu^2 - \left(\rho_y - \frac{\lambda}{\sigma^{-1}}\right)\mu - \frac{\lambda}{\beta\sigma^{-1}} = 0.$$

As in the previous section, the equilibrium is determinate if and only if the absolute value of the roots of $\mathcal{P}(\mu)$ lie on opposite sides of the unit circle.

Under the benchmark calibration, the equilibrium is determinate for values of $\lambda \in [0.093, 12.667]$. A fairly small amount of flexibility in the fiscal response to variations of real activity is sufficient to guarantee determinacy of the equilibrium. On the other hand, the feedback from the output gap should not be excessive, in order to avoid explosive debt dynamics. In this case, an exogenous shocks with inflationary consequences leads to the same interest rate increase as in the case of strict policy rules. However, the negative feedback on the government budget is now mitigated by the fiscal authority concern for real activity. Hence, the increase in taxes is now smaller than in the baseline case, triggering a more contained reaction of the monetary authority in response to the fiscal stabilization. The mechanism is qualitatively very similar to the case of flexible inflation targeting. Again, the quantitative analysis suggests that the degree of flexibility necessary to avoid indeterminacy is rather small, although generally a little stronger than for monetary policy.

A higher coefficient of risk aversion moves the interval for λ that ensures determinacy to the right with substantial skewness in favor of the high values. When $\sigma^{-1} = 1$ or 5, the interval is respectively $[0.790, 40.766]$ or $[4.099, 174.099]$. A higher inverse Frisch elasticity of labor supply widens the determinacy interval. For $\omega > 1$, determinacy occurs for any positive value of λ.

On the other hand, values of the steady state tax rate higher than 20% shrink the determinacy interval for the fiscal policy parameter λ. The thresholds are much more sensitive in this case than for flexible inflation targeting rules. If $\bar{\tau} = 30\%$, a determinate equilibrium requires $\lambda \in [0.151, 1.138]$ (see figure 3).

5 The Optimal Degree of Flexibility

The bottom line of the previous section is that flexibility in either monetary or fiscal policy helps to avoid unpleasant indeterminacy results that arise under a strict formulation of targeting rules. While this finding is very much robust to alternative parameter configurations, the average value of the tax rate appears to be critical for the design of flexible fiscal targeting rules. From this perspective, flexible inflation targeting is a more robust prescription to different parametrizations of the model. This section explores the flexibility prescription from a welfare standpoint.

Recently, Schmitt-Grohé and Uribe (2006) have shown that the combination of an aggressive inflation targeting regime with a passive fiscal policy rule closely approximates the optimal policy outcome in the context of a medium scale DSGE model. Their conclusion contrasts with the work of Benhabib and Eusepi (2005) who find that a positive response to the output gap in the Taylor rule for monetary policy can be effective in avoiding indeterminacy. Section 4 has revisited this result in the context of an inflation targeting rule. The following discussion evaluates quantitatively the welfare consequences of flexible targeting rules, adding to the debate the possibility of endowing the fiscal authority with a flexible targeting rule.

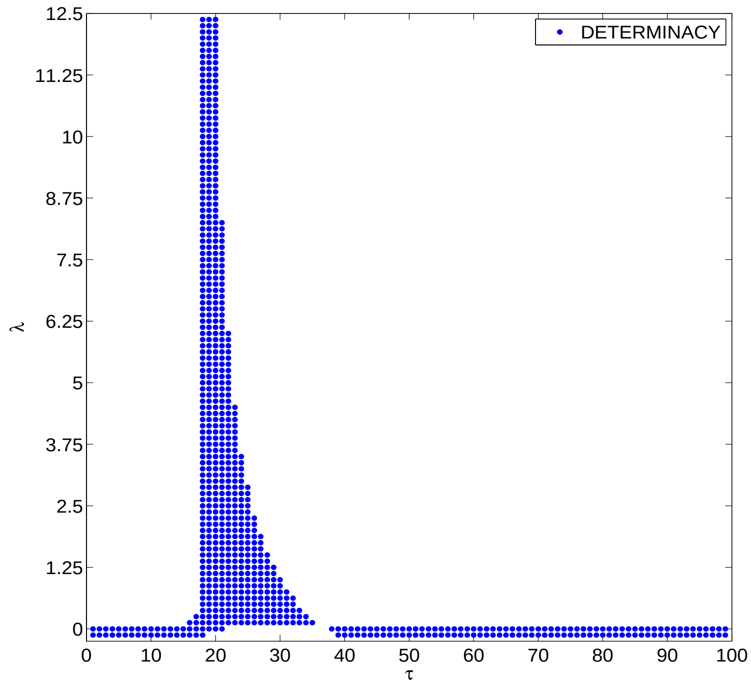

Figure 3: Flexible debt targeting and determinacy.

5.1 The Optimal Policy Benchmark

This section briefly revisits the optimal fiscal and monetary policy problem for the economy described in section 2. The objective function consists of a second order approximation to the utility of the representative agent

$$u_0 = -\frac{1}{2}\Omega E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(q_y y_t^2 + q_\pi \pi_t^2 \right) \right\} + T_0 + t.i.p. + \mathcal{O}\left(\|\xi\|^3 \right), \tag{12}$$

where T_0 is a transient component specific to time zero and $t.i.p.$ stands for "terms independent of policy".[11] The constant Ω and the weights on the output gap q_y and inflation q_π are defined in the appendix.

The approximate optimal policy problem from a timeless perspective corresponds to a linear-quadratic framework in which the benevolent policymaker chooses the entire sequence

[11] The transient component is related to the pre-commitments that are necessary to derive the optimal policy from a "timeless perspective" (Woodford, 1999).

$\{y_t,\, \pi_t,\, \hat{\tau}_t,\, \hat{b}_t\}_{t=0}^{\infty}$ to maximize [12] subject to [1] and [2] and the additional constraints that make the optimal plan time-invariant.

The first order conditions for this problem can be found following a standard Lagrangian method. Benigno and Woodford (2003) show that the optimal policy plan can be cast in terms of targeting rules which involve only endogenous variables. As discussed by Giannoni and Woodford (2002), a representation of the optimal policy in terms of targeting rules carries two advantages. First, under mild conditions, targeting rules ensure determinacy of the equilibrium.[12] Second, the optimal policy is robust to different assumptions about the nature of the stochastic process assumed for the exogenous disturbances.

The targeting rule for monetary policy is

$$\pi_t + \frac{n_\varphi}{m_\varphi}\pi_{t-1} + \frac{\omega_\varphi}{m_\varphi}\left(y_t - y_{t-1}\right) = 0, \tag{13}$$

where the coefficients m_φ, n_φ and ω_φ are combinations of the structural parameters reported in the appendix. The optimal monetary rule takes the form of a flexible inflation targeting. Expression [13] actually resembles the optimal policy rule in the baseline New Keynesian model, except for the term in lagged inflation which captures the additional degree of inertia introduced by the government budget constraint.

The fiscal targeting rule is

$$E_t \pi_{t+1} = 0. \tag{14}$$

The optimal fiscal rule commands [14] the government to set taxes without creating inflationary expectations. The interpretation of [14] as a targeting rule for fiscal policy is perhaps not so obvious. A more direct fiscal policy rule can be obtained by combining [14] with the Phillips curve [1], which yields

$$(\hat{\tau}_t - \hat{\tau}_t^*) + m_\tau y_t + n_\tau \pi_t = 0, \tag{15}$$

where m_τ and n_τ are defined in the appendix. Equation [15] indicates how the government sets taxes trying to balance movements in the output gap and the inflation rate. The most relevant fiscal aspect of the optimal policy plan is that it induces a unit root in all real variables (debt, tax gap, output gap).[13] In fact, in the special case of i.i.d. fiscal stress, debt actually follows a random walk ($\hat{b}_t = E_t \hat{b}_{t+1}$). The intuition for this result is very much in the

[12] See the appendix of Benigno and Woodford (2003) for a direct proof of determinacy of the equilibrium under the optimal targeting rules in this model.

[13] The first order conditions of the optimal policy plan (not reported here) show that the Lagrange multiplier on the government budget constraint follows a random walk.

same spirit of Barro (1979). The fiscal authority tries to smooth the impact of distortionary taxes in response to exogenous shocks affecting the fiscal stress via permanent variations in government debt.

The optimal policy plan provides a clear and natural benchmark for the evaluation of alternative rules. The next section computes the welfare costs of the simple targeting rules considered in the previous sections under different configurations of flexibility.

5.2 Optimal Flexible Targeting Rules

This section compares the optimal choice of the coefficients γ and λ in [5] and [6] with the optimal policy benchmark represented via [13] and [14]. The welfare metric adopted for the comparison is

$$d_p \equiv -\left(\frac{1-\beta}{2\bar{C}^{1-\tilde{\sigma}^{-1}}}\right)\left[E(u_0^p) - E(u_0^{opt})\right], \tag{16}$$

where u_0^{opt} represents welfare under the optimal policy plan *opt* and u_0^p stands for welfare under any alternative policy plan p consistent with the timeless perspective commitments.[14] The operator $E(\cdot)$ defines the expectation over the distribution of shocks at time zero. The welfare measure d_p is conditional on the system being in a steady state before time 0 and corresponds to a correct second order approximation of the consumption equivalent of the two policies (Lucas, 1987).[15] In other words, d_p measures the fraction of consumption under the optimal policy that the average household in the economy would be willing to give up in each period to avoid switching to regime p.

In order to compute the optimal coefficients in the policy rules and the consumption equivalents, the persistence of the fiscal stress is assumed to be equal to 0.9 and the innovations to its process to be i.i.d. standard normal. Table 2 summarizes the main results of the quantitative analysis. The first two columns report the value of the optimized coefficient and the consumption equivalent when flexibility is granted to one policy authority at the time. The third column reports the optimal coefficients and the consumption equivalent when flexibility is granted contemporaneously to both policy authorities.

The highlight of table 2 is clearly the large welfare gain of granting flexibility to the fiscal authority. On the other hand, the additional benefit of flexibility in both targeting rules is relatively small. In terms of the optimal coefficients, a flexible fiscal rule is optimal if it prescribes an aggressive response of debt to departures of output from its welfare-relevant

[14] The inclusion of the timeless perspective constraints is crucial for the appropriate welfare ranking of alternative policies. See Benigno and Woodford (2005) for details.

[15] See the extended version of Schmitt-Grohé and Uribe (2006) for the derivation.

Table 2: Optimized targeting rules coefficients (γ, λ) and consumption equivalents (d_p).

		Flexible Targeting Rule		
		Monetary	Fiscal	Both
Optimized Coefficients	γ	0.21	0	0.33
	λ	0	12.1	12.0
Consumption Equivalent	d_p	47.06%	2.93%	2.37%

target. This feature attempts to mimic the permanent variations of debt in response to a fiscal stress shock typical of the optimal policy plan.

If only the monetary authority is granted a flexible targeting rule, the weight on the growth rate of the output gap is non-negligible.[16] Interestingly, when both policy authorities are endowed with flexible targeting rules, it is optimal to even increase the weight on real activity for monetary policy. Consistently with the intuition for the indeterminacy results in the previous sections, the reason is that the fiscal stabilization carries inflationary consequences that would induce an additional monetary contraction. The optimal combination of flexible targeting rules assigns more weight to output stabilization for the monetary authority. At the same time, the monetary stabilization increases the cost to service government debt. Therefore, optimality requires a reduced relative weight on output stabilization for fiscal policy.

The general lesson of this section is that from a welfare perspective flexibility is clearly more desirable in fiscal than in monetary policy. This finding, however, partially collides with the robustness results of section 4, hence creating a tradeoff between these two concerns (welfare versus robustness).

[16] In the baseline New Keynesian model, the weight on the growth rate of the output gap coincides with the inverse of the elasticity of substitution among varieties θ. Using that result as a reference point, the coefficient γ in table 2 is at least twice as big under the calibration adopted in this paper.

6 Determinacy with Balanced Budget

This section presents an alternative specification of fiscal targeting rules that overcomes the determinacy problems discussed in section 3, even under balanced budget requirements.

The starting point is the observation that the debt variable $\hat{b}_t$ in Benigno and Woodford (2003) corresponds to the log-deviation from steady state of the real value of debt at maturity

$$\hat{b}_t \equiv \log\left(\frac{b_t}{\bar{b}}\right), \qquad \text{where } b_t \equiv \frac{R_t B_t}{P_t}.$$

Section 3 specified the fiscal rule directly in terms of $\hat{b}_t$. In particular, the constant debt rule [6] with $\lambda = 0$ requires the fiscal authority to adjust debt so that its real value at maturity remains constant in every period and state of the world. As argued above, this rule is potentially destabilizing in the sense that involves a two-way feedback between policy authorities. On the one hand, variations of the nominal interest rate directly affect the value of debt at maturity. On the other hand, fiscal policy decisions have inflationary consequences, hence requiring a response by the monetary authority. This type of fiscal-monetary policy interaction is at the very heart of the indeterminacy results discussed above.

An alternative specification of the balanced budget rule would target only the real stock of debt, without including the interest rate payments on outstanding liabilities. In this case, the flexible fiscal rule is

$$\hat{b}_t - r_t + \phi y_t = 0. \tag{17}$$

The fiscal authority then follows a balanced budget rule if $\phi = 0$ in [17].[17] This type of strict debt rule, together with a strict inflation targeting rule, leads to opposite conclusions in terms of determinacy compared to the findings of section 3. In this case, the linear model admits a closed form solution for the output gap[18]

$$y_t = -\frac{1}{\left(1-\rho_y\right)\beta\sigma^{-1}}\left(\hat{b}_{t-1} + u_{f,t} - \beta r_t^*\right). \tag{18}$$

The determinacy properties of the model depend upon the dynamics of the equation for the evolution of debt. After combining the solution for the output gap [18] with the Euler

[17] Benhabib and Eusepi (2004) and Schmitt-Grohé and Uribe (2004) also study balanced budget rule of this type in models in which the monetary authority follows a Taylor rule. See Ferrero (2008) for an application to the case of a currency union.

[18] As usual, I substitute the monetary policy rule into the Phillips curve and express the tax gap as a function of the output gap. I plug the result into the government budget constraint and combine it with the fiscal policy rule.

equation [4] and the fiscal rule [17], it is possible to see that the solution for debt is

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_{b,t}, \tag{19}$$

where

$$\rho_b \equiv \frac{1}{1 + \beta \left(1 - \rho_y\right)}$$

and

$$\varepsilon_{b,t} \equiv \rho_b \left[\left(u_{f,t} - E_t u_{f,t+1}\right) - \beta \left(\rho_y r_t^* - E_t r_{t+1}^*\right) \right].$$

The dynamics of debt, and hence the determinacy properties of the model, are governed by the coefficient ρ_b. In particular, the equilibrium is determinate if and only if $|\rho_b| < 1$.

Under the benchmark calibration, $\left|\rho_y\right| < 1$. Therefore, the equilibrium under strict inflation targeting rule and the strict debt targeting rule in [17] with $\phi = 0$ is determinate. Moreover, the determinacy regions as a function of the steady state tax rate are exactly the opposite with respect to the case of section 3. Figure 4 shows that determinacy occurs for intermediate values of steady state tax rate $\bar{\tau} \in [18.7\%, 38.6\%]$.[19]

The intuition for why the equilibrium is determinate under the strict formulation of the targeting rules [5] and [17] is that the modified fiscal rule does not respond to movements in the nominal interest rate. In this case, when the monetary authority increases the nominal interest rate in response to an inflationary shock, there is no direct feedback for the government budget. Clearly, the fiscal balance is influenced by the general equilibrium effects on output and inflation. But these effects are far from unambiguous. Indeed, in the absence of a monetary policy reaction, an inflationary shock would be beneficial for the government budget to the extent that (i) it boosts revenues by increasing output and (ii) it reduces the real value of government liabilities by increasing inflation. The monetary policy reaction counterbalances these two effects.

Given that rule [17] induces determinacy under strict inflation targeting when $\phi = 0$, it seems natural to ask whether its flexible version ($\phi > 0$) is robust to alternative parameter configurations and steady state fiscal stances. In this case, the solution for debt is again a first order autoregressive process

$$\hat{b}_t = \varrho_b \hat{b}_{t-1} + \epsilon_{b,t}, \tag{20}$$

where

$$\varrho_b \equiv \frac{1 + \phi/\sigma^{-1}}{1 + \beta \left[\left(1 - \rho_y\right) + \phi/\sigma^{-1} \right]}$$

[19] The robustness analysis to different values of the coefficient of risk aversion and the inverse of the Frisch elasticity of labor supply coincides with the results of section 3.

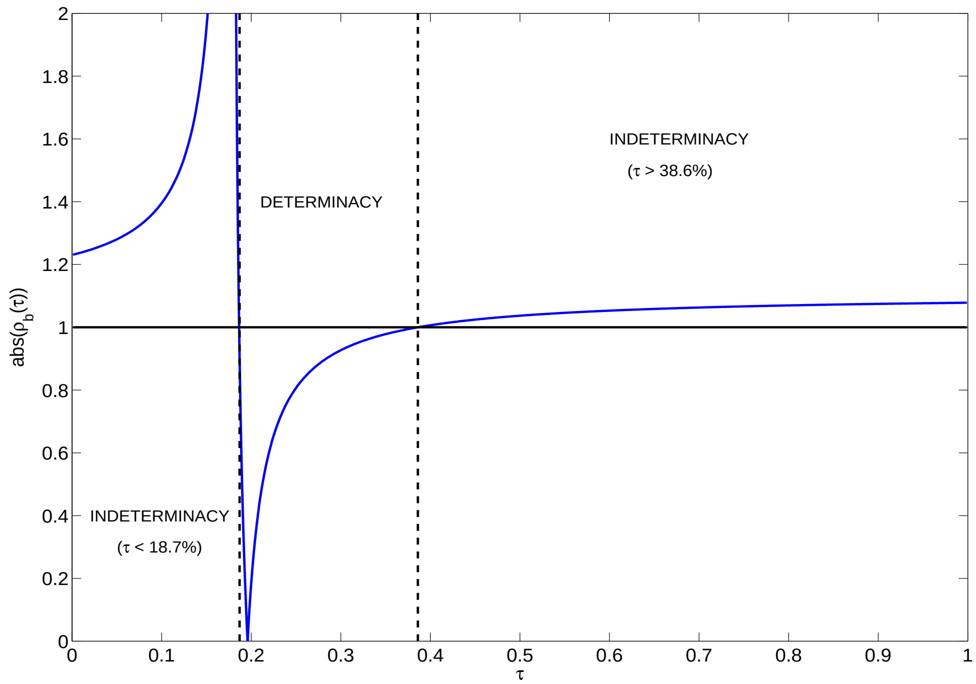

Figure 4: Absolute value of ρ_b as a function of $\bar{\tau}$.

and

$$\epsilon_{b,t} \equiv \left(\frac{\varrho_b}{1+\phi/\sigma^{-1}}\right)\left\{\left[\left(1+\phi/\sigma^{-1}\right)u_{f,t} - E_t u_{f,t+1}\right] - \beta\left(\rho_y r_t^* - E_t r_{t+1}^*\right)\right\}.$$

Clearly, if $\phi = 0$, [19] and [20] coincide since $\varrho_b = \rho_b$ and $\varepsilon_{b,t} = \epsilon_{b,t}$.

Determinacy requires $|\varrho_b| < 1$. Since under the baseline calibration $\left|\rho_y\right| < 1$, indeterminacy can arise only for values of ϕ high enough (again the threshold is 12.667). The robustness analysis to variations in the coefficient of risk aversion and the inverse Frisch elasticity of labor supply essentially coincides with the previous formulation of flexible debt targeting rules as far as the upper threshold for ϕ is concerned. The key difference is that the lower threshold is not influenced by changes in the parameter values. Balancing the budget via a strict version of rule [17] is always a viable possibility.

Figure 5 portraits the determinacy region as a function of $\bar{\tau}$ for when ϕ is positive. Flexible debt targeting remains feasible for very much the same range of steady state tax rates associated with determinacy under balanced budget.

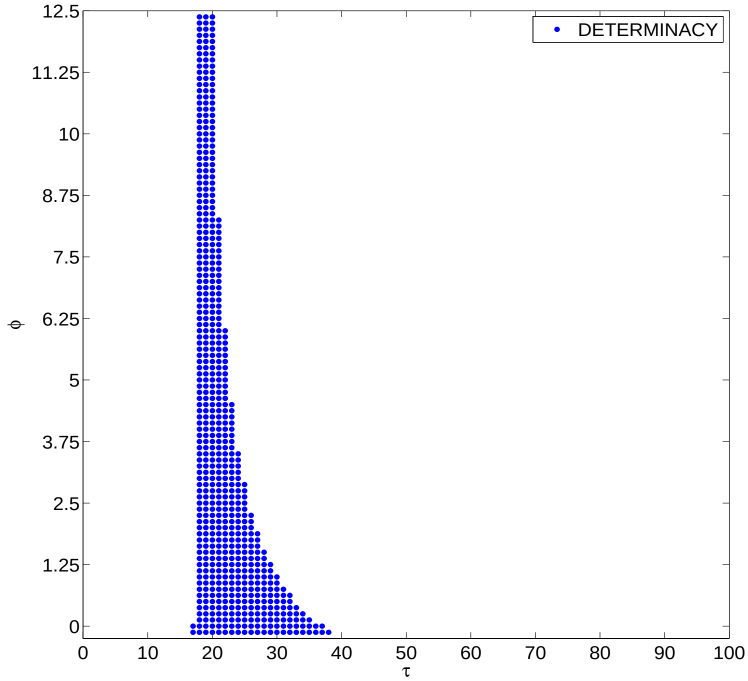

Figure 5: Flexible debt targeting redux and determinacy

The last question that this section addresses then is how much flexibility is actually desirable in the modified fiscal targeting rule considered here. Clearly, the answer also bears consequences for the optimal degree of flexibility in monetary policy. Table 3 reports the quantitative results in the same format as in the previous section.

One preliminary observation is in order. The consumption equivalent numbers in tables 2 and 3 are not directly comparable. The solution of the model under the modified flexible debt targeting rule [17] requires the specification of a stochastic process for the equilibrium real interest rate r_t^*. For simplicity, this variable is assumed to follow an first order autoregressive process with persistence equal to 0.9 and i.i.d. standard normal innovations. This assumption adds an independent source of exogenous variability relative to the fiscal stress and increases the overall costs of business cycle fluctuations.[20] Since the emphasis here is on the gains from fiscal relative to monetary flexibility, the simplification of making the two processes

[20]In Benigno and Woodford (2003), r_t^* and f_t are instead functions of the same fundamental shocks and hence correlated.

Table 3: Optimized targeting rules coefficients (γ, ϕ) and consumption equivalents (d_p).

		Flexible Targeting Rule		
		Monetary	Fiscal	Both
Optimized Coefficients	γ	0.18	0	0.02
	ϕ	0	12.6	12.6
Consumption Equivalent	d_p	170.6	3.895	3.856

independent of each other does not affect the main result.

As in the previous section, the gains of granting flexibility to the fiscal relative to the monetary authority are very large, indeed close to 100%. On the other hand, the gains of combining flexibility in both forms of policy relative to flexible fiscal targeting rules only are quite modest, of the order of 1%. These results stress even further the superior welfare properties of flexible fiscal rules.

The second element worth mentioning about table 3 concerns the optimal value of ϕ. In all simulations, optimality pushes the coefficient to the determinacy threshold. The reason is that the flexible targeting rule tries to approximate the unit root in debt typical of the optimal policy plan. Moreover, as evident from the consideration above about welfare, flexibility in fiscal policy almost mutes the necessity of flexibility in monetary policy. The optimal coefficient in the inflation targeting rule decreases from 0.18 to 0.02 when the fiscal authority is allowed to respond to variations in the output gap rather than balancing the budget in every period. The optimized coefficient in the flexible debt targeting rule is unchanged independently of whether the monetary authority pursues a strict or an optimized inflation targeting rule. This outcome contrasts with the findings in table 2, in which the optimized inflation targeting rule features a higher coefficient when the fiscal authority follows a flexible debt targeting rule.

The bottom line of this section is that the indeterminacy result of section 3 is sensitive to the specification of the balanced budget rule. From a welfare perspective, flexibility in fiscal policy remains highly desirable. Nevertheless, if reasons that go beyond the stabilization motive analyzed in this paper call for a strict formulation of fiscal and monetary targeting rules, this section suggests that the appropriate design of such rules should avoid a perverse feedback effect from monetary policy onto fiscal solvency via the nominal interest rate.

7 Conclusions

The point of departure of this paper was to show that the combination of strict inflation targeting and balanced budget rules might give raise to indeterminate equilibria for a wide range of parameter configurations and steady state tax rates. The key reason for this indeterminacy result is the spiral feedback between monetary and fiscal policy. The monetary contraction in response to an inflationary shock leads the fiscal authority to raise the tax rate in order to finance the higher debt service and maintain a balanced budget. In turn, however, higher taxes have inflationary consequences that the monetary authority seeks to counteract with a further hike in the interest rate. In the absence of an offsetting mechanism, this loop can lead to indeterminacy.

The rest of the analysis demonstrated that the indeterminacy problem can be overcome if either the fiscal or monetary authority introduces a concern for stabilization of the output gap in its targeting rule. The intuition is that augmenting the targeting rules with a concern for real activity balances the policy response to an exogenous shock in the opposite direction relative to the main stabilization objective (inflation and debt). The quantitative analysis suggests that flexible inflation targeting is a more robust prescription for solving the indeterminacy problem. On the other hand, an optimal flexible debt targeting rule is likely to have more desirable welfare properties.

The last section proposed an alternative balanced budget rule that prevents indeterminacy to occur even in the presence of strict inflation targeting. This formulation requires the fiscal authority to stabilize debt net of interest rate spending. Such a rule eliminates the feedback of the monetary policy action onto fiscal policy via the nominal interest rate. As a consequence, the fiscal stabilization operates only in response to the exogenous shock and hence limits the intensity of the additional monetary reaction.

The general message of the paper is that considering the design of fiscal and monetary policy rules in isolation might lead to unpleasant equilibrium outcomes. The consequences of monetary actions for fiscal policy decisions (and vice versa) should be carefully evaluated in the context of a model that allows for both to be relevant.

References

[1] Barro, R. (1979), "On the Determination of Public Debt," *Journal of Political Economy* 87, pp. 940-971.

[2] Benhabib, J. and S. Eusepi (2005), "The Design of Monetary and Fiscal Policy: A Global Perspective," *Journal of Economic Theory* 123, pp. 40-73

[3] Benigno, P. and M. Woodford (2003), "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," in *National Bureau of Economic Research Macroeconomics Annual 2003,* M. Gertler and K. Rogoff (eds.), MIT Press – Cambridge, MA.

[4] Bernanke, B., Laubach, T., Mishkin, F. and A. Posen (1999), *Inflation Targeting. Lessons from the International Experience*, Princeton University Press – Princeton, NJ.

[5] Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12, pp. 383-398.

[6] Clarida, R., Galí, J. and M. Gertler (1999), "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature* 37, pp. 1661-1707.

[7] Evans, G. and S. Honkapohja (2006), "Monetary Policy, Expectations and Commitment," *Scandinavian Journal of Economics* 108, pp. 15-38.

[8] Ferrero, A. (2008), "Fiscal and Monetary Rules for a Currency Union," *Manuscript*, Federal Reserve Bank of New York.

[9] Galí, J., Gertler, M. and D. López-Salido (2007), "Markups, Gaps, and the Welfare Cost of Economic Fluctuations," *Review of Economics and Statistics* 89, pp. 44-59.

[10] Giannoni, M. and M. Woodford (2002) "Optimal Interest-Rate Rules: I. General Theory," *National Bureau of Economic Research Working Papers* No. 9419.

[11] Leeper, E. (1991), "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics* 27, pp. 129-147.

[12] Schmitt-Grohé, S. and M. Uribe (1997), "Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability," *Journal of Political Economy* 105, pp. 976-1000.

[13] Schmitt-Grohé, S. and M. Uribe (2006), "Optimal Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics* 54, pp. 1702-1725.

[14] Svensson, L. (2003), "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules," *Journal of Economic Literature* 41, pp. 426-477.

[15] Svensson, L. (2007), "Inflation Targeting," in *The New Palgrave Dictionary of Economics* (2nd edition), L. Blum and S. Durlauf (eds.), Palgrave Macmillan – New York, NY.

[16] Taylor, J. (1999), "A historical analysis of monetary policy rules," in *Monetary Policy Rules*, J. Taylor (ed.), The University of Chicago Press – Chicago, IL.

[17] Trabandt, M. and H. Uhlig (2006), "How Far Are We From the Slippery Slope? The Laffer Curve Revisited," *Centre for Economic Policy Research Discussion Papers* No. 5657.

[18] Woodford, M. (1999), "Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?" in *New Challenges for Monetary Policy*, Federal Reserve Bank of Kansas City.

[19] Woodford, M. (2003), *Interest and Prices. Foundations of a Theory of Monetary Policy*, Princeton University Press – Princeton, NJ.

A The Economy

Household $j \in (0,1)$ values consumption C_t^j and dislikes hours worked ℓ_t^j according to the utility function

$$u_0^j \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^j)^{1-\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}} - \frac{(\ell_t^j)^{1+v}}{1+v} \right] \right\},$$

where β is the discount rate. The consumption index is

$$C_t^j \equiv \left[\int_0^1 c_t^j (i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where $c_t^j(i)$ is the consumption of good i by household j whose price is $p_t(i)$. The flow budget constraint is

$$\int_0^1 p_t(i)\, c_t^j(i)\, di + E_t\{Q_{t,t+1} D_{t+1}^j\} = w_t^j \ell_t^j + \Gamma_t^j + D_t^j,$$

where D_{t+1}^j denotes the payoff of a portfolio of state-contingent securities purchased by household j at time t whose price is $Q_{t,t+1}$. The nominal wage w_t^j, is household-specific as a consequence of the assumption of labor market segmentation. Finally, the variable Γ_t^j stands for after-tax nominal profits from ownership of the firms.

Firm $i \in (0,1)$ hires workers and produces according to the technology

$$y_t(i) = a_t \ell_t(i)^{\frac{1}{\phi}},$$

where a_t is an economy-wide technology shock. Each firm acts as a wage-taker in segmented labor markets. Prices are assumed to be set on a staggered basis (Calvo, 1983). The time T profit function of firm i is

$$\Gamma_{t,T}(i) = (1-\tau_T)\, p_t(i)\, y_{t,T}(i) - w_T(i)\, \ell_T(i),$$

where $t \leq T$ is the last period of price adjustment, τ_T is the sales tax rate (the fiscal instrument) and $y_{t,T}(i)$ is the demand of good i at time T conditional on the price of good i not being changed since period t.

The monetary policy instrument is the nominal interest rate R_t. Following Woodford (2003), the model abstracts from monetary frictions and considers the limit of a "cashless economy".

The flow government budget constraint is

$$B_t = R_{t-1}B_{t-1} - \int_0^1 p_t(i)\left[\tau_t y_t(i) - g_t(i)\right] di - \varsigma_t,$$

where B_t represents government debt, $g_t(i)$ is the amount of government spending on the generic variety i and the term ς_t denotes lump-sum transfers which the government takes as exogenous.[21]

A.1 Equilibrium and the Optimal Policy Problem

The optimal policy problem consists of maximizing

$$u_0 = E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\tilde{\sigma}^{-1}}}{1-\tilde{\sigma}^{-1}} - \frac{(Y_t/a_t)^{1+\omega}}{1+\omega}\Delta_t\right]\right\},$$

where

$$\Delta_t \equiv \int_0^1\left[\frac{p_t(i)}{P_t}\right]^{-\theta(1+\omega)} di,$$

subject to:

1. The Euler equation

$$1 = \beta R_t E_t\left\{\frac{1}{\Pi_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\tilde{\sigma}^{-1}}\right\},$$

   where $\Pi_t \equiv P_t/P_{t-1}$ defines the inflation rate and the price index is given by

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}.$$

2. The Phillips curve

$$\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1+\theta\omega}{\theta-1}} = \frac{F_t}{K_t},$$

   where

$$F_t = (1-\tau_t)\,C_t^{-\tilde{\sigma}^{-1}}Y_t + \alpha\beta E_t\left\{\Pi_{t+1}^{\theta-1}F_{t+1}\right\},$$

[21] The government chooses optimally how to allocate a given (exogenous) amount of total expenditure G_t among existing varieties. The aggregator for public spending has the same functional form as the consumption index.

$$K_t = \left(\frac{\theta}{\theta - 1}\right) \mu_t^w \left(\frac{Y_t}{a_t}\right)^{1+\omega} + \alpha\beta E_t\{\Pi_{t+1}^{\theta(1+\omega)} K_{t+1}\}$$

and total output is

$$Y_t = C_t + G_t.$$

3. The evolution of the index of price dispersion

$$\Delta_t = \alpha\Delta_{t-1}\Pi_t^{\theta(1+\omega)} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\omega)}{\theta-1}}.$$

4. The government budget constraint

$$\frac{C_t^{-\tilde{\sigma}^{-1}} b_{t-1}}{\Pi_t} = C_t^{-\tilde{\sigma}^{-1}} (\tau_t Y_t - G_t - \varsigma_t) + \beta E_t \left\{ \frac{C_{t+1}^{-\tilde{\sigma}^{-1}} b_t}{\Pi_{t+1}} \right\},$$

where

$$b_t \equiv \frac{R_t B_t}{P_t}.$$

The linear-quadratic optimal policy problem discussed in the text represents a correct approximation, up to the second order, to the non-linear optimal policy problem presented in this section.

B Definition of the Parameters

The deep parameters of the model are the discount factor β, the risk aversion coefficient $\tilde{\sigma}^{-1}$, the inverse Frisch elasticity ν, the curvature of the production function ϕ, the probability of not adjusting the price α and the elasticity of intratemporal substitution θ.

The steady state consumption-to-output ratio is defined by s_C and the steady state spending-to-output ratio is defined by s_G, so that the resource constraint can be written as

$$s_C + s_G = 1.$$

On the other hand, the steady state tax rate is $\bar{\tau}$, the steady state debt is $\bar{b}$ and the steady state output is $\bar{Y}$, so that the government budget constraint can be written as

$$s_d \equiv (1-\beta)\frac{\bar{b}}{\bar{Y}} = \bar{\tau} - s_G.$$

The steady state wage markup shock is denoted by $\bar{\mu}^w$ and the steady state productivity parameter by $\bar{a}$. With monopolistic competition, steady state output is determined by

$$\bar{Y} = \left[(1-\Phi)\, \bar{a}^{1+\omega} \left(1 - s_G\right)^{-\tilde{\sigma}^{-1}} \right]^{\frac{1}{\omega+\tilde{\sigma}^{-1}}},$$

where the measure of steady state output inefficiency is given by

$$\Phi \equiv 1 - \left(\frac{\theta - 1}{\theta}\right)\left(\frac{1-\bar{\tau}}{\bar{\mu}^w}\right) < 1.$$

The remaining composite parameters are

$$\begin{aligned}
\sigma^{-1} &\equiv \tilde{\sigma}^{-1} s_C^{-1} \\
\omega &\equiv (1+\nu)\,\phi - 1 \\
\omega_\tau &\equiv \frac{\bar{\tau}}{1-\bar{\tau}} \\
\omega_g &\equiv \frac{s_G}{s_d} \\
b_\tau &\equiv 1 + \omega_g \\
b_y &\equiv b_\tau - \sigma^{-1} \\
\psi &\equiv \frac{\omega_\tau}{\omega + \sigma^{-1}} \\
\kappa &\equiv \frac{(1-\alpha)(1-\alpha\beta)\left(\omega+\sigma^{-1}\right)}{\alpha\,(1+\theta\omega)}.
\end{aligned}$$

Finally, the parameters of the second order approximation of the loss function are defined as

$$\begin{aligned}
\Omega &\equiv s_C^{-\tilde{\sigma}^{-1}} \bar{Y}^{1-\tilde{\sigma}^{-1}} \\
\Gamma &\equiv \left(\omega + \sigma^{-1}\right) b_\tau - \omega_\tau b_y \\
\Lambda &\equiv \left(\omega + \sigma^{-1}\right)\left[(1-\Phi) + \frac{\Phi\,(1+\omega)\, b_\tau}{\Gamma}\right] \\
q_y &\equiv \Lambda + \frac{\Phi\sigma^{-1}}{\Gamma}\left[(1+\omega_\tau)\, b_\tau - s_C^{-1}\left(b_\tau + \omega_\tau\right)\right] \\
q_\pi &\equiv \frac{\theta\Lambda}{\kappa}.
\end{aligned}$$

C Roots of a Second Order Difference Equation

Proposition 1 *Let $P(\lambda) \equiv \lambda^2 + A_1\lambda + A_0 = 0$ and let λ_1 and λ_2 be the roots of $P(\lambda)$. Then, the absolute values of λ_1 and λ_2 split across the unit circle if and only if $P(1) > 0$ and $P(-1) < 0$ or vice versa.*

Proof. Notice that one can always rewrite the polynomial $P(\lambda)$ as

$$P(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \tag{21}$$

1. First, show that $P(1) > 0$ and $P(-1) < 0$ imply that the absolute values of the two roots λ_1 and λ_2 split across the unit circle.

   From the right hand side of [21], it is easy to see that

   $$P(1) = (1 - \lambda_1)(1 - \lambda_2) > 0, \tag{22}$$

   and

   $$P(-1) = (1 + \lambda_1)(1 + \lambda_2) < 0. \tag{23}$$

   If $P(1) > 0$, it means that λ_1 and λ_2 are on the same side of 1. Similarly, if $P(-1) < 0$, it means that λ_1 and λ_2 are on opposite sides of -1. It then follows that one root must lie inside the unit circle and the other outside. The case $P(1) < 0$ and $P(-1) > 0$ is totally symmetric.

2. Next, show that if $|\lambda_1|$ and $|\lambda_2|$ lie on opposite sides of 1, it must be the case that $P(1)$ and $P(-1)$ lie on opposite sides of 0.

   Without loss of generality, suppose $|\lambda_1| > 1$ and $|\lambda_2| < 1$. There are two cases two be considered. First, if $\lambda_1 > 1$, then, one can see from [22] that $P(1) < 0$ and from [23] that $P(-1) > 0$, which confirms the claim. Second, if $\lambda_1 < -1$, then, again from [22] and [23], $P(1) > 0$ and $P(-1) < 0$. The case $|\lambda_1| < 1$ and $|\lambda_2| > 1$ is symmetric.

■

Proposition 1 complements Proposition $C.1$ in Woodford (2003) which gives necessary and sufficient conditions for the two roots of $P(\lambda)$ to be both outside the unit circle.