# Academic and Industrial R&D, Heterogeneous IPR, and Growth

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**Abstract**

The empirical evidence show that academic and industrial research are strongly connected. With the aim to enhance the technological transfer from academia to industry, the policy design of many OECD countries allows worth ideas obtained in academia, and in other government funded and non-profit organizations, to be granted of same intellectual property rights (IPR) as for industrial innovations. Yet, the empirical analyses show the existence of both a substitutability and complementarity relationship between industrial and public R&D investments. In distinguishing the stages of a R&D process within a dynamic general equilibrium model, this paper explicitly considers both industrial and government R&D efforts. The theoretical results show that a high enough population growth rate as compared to institutional parameters, allows a complementarity relationship between industrial and public R&D investments to emerge. Such a complementarity relationship allows the per capita output growth rate to be higher. Moreover, the results show that a 'softer' IPR regime granted to academic ideas than to industrial innovations increases the per capita growth rate of the economy.

Keywords: Industrial and Academic R&D, Intellectual Property Rights, Growth.

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# 1 Introduction

The role of both scientific knowledge and technological innovations to spur the growth performance of countries is emphasized by both academic and non-academic analyses. The OECD Science, Technology and Industry Outlook (2006) shows that real expenditures on R&D increased by more than one-third between 1995 and 2004 across the OECD, with an average annual increases of 3.8%. The effort in knowledge accumulation from government and public authorities is also high in most developed economies. The US and the EU-25 doubled their academic R&D expenditures from 1990 to 2003.[1] Moreover, the empirical evidence shows that - along a R&D process - the basic stage is strongly financed by the public sector, and it is conducted in non-profit organizations such as academia, while the development stage is the prominent activity of the industrial research effort.[2]

For the industrial sector, firms' R&D expenditures represent a fixed cost with a fundamental uncertainty and serendipity for the offsprings of their effort. Therefore, firms need to gain some rents once successful in research. A traditional legal instrument intended to provide such an economic incentive consists in granting intellectual property rights (IPR) to industrial innovations. Moreover, politicians, and public institutions in general, based their IPR policy design to enhance the transfer of knowledge from universities to industry, the commercialization of academic ideas, and the academic contribution to innovation and

[1] In the major OECD regions, percentage-point gains were largest in Japan, which jumped from 2.9% to 3.13%, followed by the United States, which increased from 2.51% to 2.68% between 1995 and 2004, and then the EU25, which saw a more modest increase from 1.69% to 1.81% between 1995 and 2003. Among the full set of OECD member countries, the largest gains were in Iceland, Finland, Denmark, Austria and Sweden, each of which added between 0.6 and 1.4 percentage points to their total R&D intensity. The SEI (2006) states that the "U.S. and the EU-25 (including 10 new member countries) have been spending similar amounts for academic R&D, from $41 to $44 billion in 2003...OECD nations other than the United States spent $74 billion, an increase of 120% over 1990. However, China has experienced the most rapid growth in its spending for academic R&D, from $1.1 billion in 1991 to $7.3 billion in 2002, with double-digit growth rates since 1999."

[2] The SEI (2006, ch.4 p.13)states: "The development of new and improved goods, services, and processes is dominated by industry, which performed 90.2% of all U.S. development in 2004...The federal government, estimated to have found 61,8% of U.S. basic research in 2004, has historically been the primary source of support for basic research...Industry devoted only an estimated 4,8% of its total R&D support to basic research in that year." See also Pavitt (2001). In the U.S. - as in other developed countries - universities are also private and non-profit institutions. However, as the data point out, the government has historically been the main financial source for basic and scientific knowledge accumulation. The Science and Engineering Indicators (SEI, 2006) by the National Science Foundation (NSF) defines as basic the research aimed "to gain more comprehensive knowledge or understanding of the subject under study without specific application in minds". The development stage is defined as "the systematic use of the knowledge or understanding gained from research directed toward the production of useful materials, devices, systems, or methods, including the design and development of prototypes and processes." An essential feature of this model is that academia only conducts early-stage of research, i.e. basic research. This is line with the empirical evidence, with Howitt's (2003) analysis, and with the theoretical results of Aghion, Dewatripont, and Stein (2005). In this paper industry is allowed to conduct basic, applied, and development stages of a R&D process.

growth. To fix ideas, the SEI (2006) documents that "also academic R&D has seen robust growth in many countries as governments try to stimulate basic research capability and to connect universities with industry for the efficient exploitation of research results." To this aim, since the '70s, the U.S. extended IPR protection to research tools and basic ideas in general, i.e. to new knowledge that does not have an immediate and specific market application. As an example, the Bayh-Dole Act of 1980 was meant to stimulate the commercialization of research by small firms and academic organizations.[3] Similar legislation is being considered in other OECD countries and it was introduced, among others, in EU, Japan, Australia (see OECD Science, Technology and Industry Outlook, 2006). The existence of strong links between academia and industry is well documented, among others, by large and direct spillovers of university's basic ideas on industry. To fix ideas, Narin *et al.* (1997), and McMillan *et al.* (2000) show that, for the U.S. industry, relying in external sources of knowledge centers on public science. Toole (1999) shows that the stock of public basic knowledge generates positive spillovers on industry. Moreover, Adams (2000) finds that spillovers the stock of federally funded academic knowledge on industry are highly specific, in that firms' academic learning responds strongly to federally funded R&D in closely affiliated universities.[4]

Yet, the empirical analyses do not agree on the existence of a complementarity/substitutability relationship between public and industrial R&D investments. David *et al.* (2000) also find the existence of large and direct spillovers from public basic knowledge to industry, but they remark that empirical evidence on public research as complement or substitute for private research is not conclusive. Indeed, the same authors maintain: "...available empirical evidence on the issue remains rather short of being conclusive, to say at least."[5]

Up to now little theoretical attention - within a dynamic general equilibrium framework - is paid to analyze the knowledge links between academia and industry, and their effect on the long run growth performance of a country.

To this aim, a multisector quality ladder model with growing population in the spirit of Aghion and Howitt (1998) is adopted. This is done because, as will

[3] Other legislative acts in such direction are the Stevenson-Wydler Technology Innovation Act of the (1980), the Small Business Innovation Development Act (1982), the National Cooperative Research Act (1984), the Federal Technology Transfer Act (1986), the National Cooperative Research and Production Act (1993), the Technology Transfer and Commercialization Act (2000).

[4] See, among others, Mokyr (2002), Nelson (2004). In particular, Narin *et al.* (1997) show that during the 1993-1994, 73% of the scientific paper cited by U.S. industrial patents were firm public science sources. Toole (1999) finds that the stock of public research stimulates private R&D investments only after a lag, with an elasticity of private R&D investment with respect to the stock of public basic research in the range of 0.46-0.53. Adams (2000) studies the effect of federally funded academic R&D on both private research effort and patenting. The author shows that the full effect of both exogenous and endogenous spillovers exceeds the structural effect that treats industrial innovation as a function of firm R&D and spillovers.

[5] Von Tunzelmann and Martin (1998) considers R&D time series for 22 OECD countries over the period 1969-1995. They find a significant and positive impact of government funded R&D on industry funded R&D in only 7 of 22 countries in their sample. Other macro empirical works that show the existence of complementarity between publicly funded R&D and private R&D are Levy (1990), Robson (1993), Diamond (1998), Leyden and Link (1991).

be shown below, the population growth rate plays a crucial role in both results and policy implications. As in Cozzi and Galli (2007), the distinction of different R&D stages is considered, with a full IPR protection assumed at all stages of the R&D process. In this set up, skilled workers endogenously decide to allocate their labor effort in a manufacturing sector, and in a R&D sector. This sector is composed of both academic and industrial R&D laboratories. In particular, and differently from the existing literature, the role of the public sector is explicitly accounted for in that government pays skilled workers to conduct basic research programs in academia. Because of a full IPR protection at all stages of a R&D process, the industry pays to use such protected academic ideas. Moreover, the government pays an undifferentiated subsidy to the industrial R&D effort.

This paper is a first attempt to shed light on the mixed empirical evidence about a substitutability-complementarity relationship between public and industrial R&D investments. It is shown that higher public R&D expenditures can either crowd out or complement industrial R&D effort depending on a threshold for the population growth rate pinned down by both structural and institutional parameters. A complemetarity relationship between government and industrial R&D investments is obtained when population grows at a high enough pace. Such a complementarity relationship allows the per capita output growth rate to be increased. Instead, if population growth rate is not high enough, higher public R&D expenditures divert too much resources from the private sector - both manufacturing and R&D - and it crowds out the industrial R&D investment. Such results show that a policy that affects the net fertility rate of a country, also affects the R&D effort effectiveness, and the per capita output growth rate. Finally, the existence of a trade-off between the per capita output growth rate and level is found. This trade-off is stronger when a complementarity relationship between private and public R&D exists.

Moreover, the effect of a same IPR protection at all stages of the R&D process is analyzed. It is shown that either a higher subsidy to industrial R&D cost or a lower cost to use academic basic ideas - that is a 'softer' IPR protection for academic ideas - increase both the industrial innovative effort and the per capita output growth rate, while they reduce the per capita output level. Such results have a main policy implication. Once IPR protection is granted to basic knowledge and research tools, a heterogeneous IPR regime should be set, with a tighter IPR protection granted to industrial innovations than to academic ideas. Indeed, the existence of IPR protection for academic knowledge generates a further cost for the industry, such as that for licenses. Importantly, this different regime for academic and industrial sectors would not restrict the technological transfer from universities to industry because of the diffuse use of informal channels to access academic ideas. Indeed, in analyzing the public basic spillovers on industry, Cohen *et al.* (2002) find that the most important channels to access publicly funded research are publications, conferences, informal interactions rather than more institutional channels such as patents, licenses, and cooperative ventures. These results refer to all industrial sectors, even the high-tech industries. Adams (2000) finds that informal learning plays a central role in generating endogenous spillovers.[6] Therefore, the diffuse use of informal channels to access public science shows that knowledge spillovers from academia to industry are not necessarily tied to a full IPR protection for earlier stages of a R&D process. Moreover, although academia can be thought as a second-best solution to the underinvestment problem caused by insufficient appropriability,[7] some authors argue an anti-commons effect associated with early IPR protection because this policy can stifle innovations downstream in the course of research and development, especially in biotechnology and pharmaceuticals (see Heller and Eisenberg, 1998).

The rest of the paper is organized as follows. Section 2 ties the paper with the existing literature. Section 3 sets up the model. Section 4 describes the general equilibrium results. Section 5 studies the comparative statics results. Section 6 draws conclusions.

# 2 Related Literature

The paper relates to a number of different literatures. The analysis concerns the formal mechanisms to protect firms' knowledge and innovations such as IPR protection. In such a way, the paper allows a comparison of IPR protection granted at different stages of a R&D process to be carried out within a dynamic general equilibrium framework.[8]

Recently, Cozzi and Galli (2007) focus on the sequential nature of the innovation process within a dynamic general equilibrium framework. The authors show that a pro-growth policy consisting into guarantee an intellectual property protection for 'basic half-ideas' could not be at the ground of the reforms undertaken in the U.S. around the '80s. This paper focuses on the strength of IPR, once such a protection is granted at all stages of the R&D process. Therefore, this paper complements Cozzi and Galli's (2007) contribution. Aghion and Howitt (1996) also distinguish the stages of a R&D process, with industry that invests in fundamental (basic) research whereby the number of product lines increases over time. O'Donoghue and Zweimuller (2004) extend the patent-design literature to a general equilibrium framework. Their results show that a tighter patent protection spurs the industrial innovative effort, and therefore the per capita output growth rate. Their work considers industrial innovations, and does not consider neither the role of public R&D investment nor different stages of a R&D process. As in such a related literature, because of the macroeconomic

[6] Moreover, the rapid increase in academic patenting has provoked new debates about the quality of these patents, and about a industry-biased university research targets induced by the stronger influence of the industry on scientific research. The existence of such a bias would reinforce the results of the model and its policy implications. See D'Este and Patel (2007).

[7] Nelson (1959), and Arrow (1962) argue that because of knowledge spillovers and imperfect IPR protection, the economic value of some ideas can not be fully appropriated by the developers, leading to private sector underinvestment in basic research. This is a traditional argument in favor of publicly funded basic research programs.

[8] Cohen *et al.* (2000) find that in 6 on 10 final good sectors firms consider the legal and formal mechanisms more effective to protect innovations (jointly patent and other legal) than non-legal mechanisms such as secrecy and lead time.

framework, this paper does not touch the microeconomics motivation in firms' patent decisions.[9]

# 3 The Model Set-up

The economy is composed of a final good sector, of an intermediate good sector, and of a R&D sector. As in Aghion and Howitt (1998), perfectly competitive firms produce a homogeneous final consumption good by combining a constant input, and a continuum of intermediate goods with heterogeneous productivity. As new intermediate goods are patented, manufacturing firms have a local monopolistic power. In such a set-up I refer indifferently to patent and IPR protection. This is done because what matters for the analysis is the tightness of a same IPR protection at different stages of a R&D process.

The R&D sector is composed of both industrial and public (academia) research units. The public sector employes skilled workers to obtain basic knowledge and research tools.[10] According to the legislative acts mentioned in the introduction, basic knowledge and research tools are granted of a full IPR protection as industrial innovations. The stock of basic ideas can be usefully developed by industry to find new commercial valuable innovations. Therefore, the industrial research sector combines academic basic ideas with its R&D effort - that can consists in the basic, applied, and development stages - to upgrade the quality (or the production process) of any intermediate product line. As said above, a perfectly enforceable patent law allows industrial research firms to gain monopolistic rents for all the effective duration of the patent because - as usual in Schumpeterian growth models with quality-ladder innovations - the incumbent monopolist can be replaced by the next innovator along the same product line.[11] Indeed, each monopoly is challenged by outsider R&D firms trying to invent and patent a better product that - due to instantaneous price competition - drives the former monopolist out of the market. This generates

[9] In particular, secrecy, lead time, and complementary assets are considered important mechanisms to protect firm's innovations and strategic knowledge (see Levin et al. 1987, Cohen *et al.* 2000). Moreover, several studies show that firms patent for many different reasons such as signaling devices, bargaining chips, or as a way to preclude entry in a given product market space. Recently, it is shown that also academic researchers patent for many reasons such as to publicly signal one's claim over a certain type of research, and call for collaborations (see Breschi and Lissoni, 2002; Murray, 2004; Franzoni and Lissoni; 2006; Forti *et al.*, 2007).

[10] In the U.S. - as in other developed countries - universities are also private institutions. However, as indicated in the Introduction, the government has historically been the main financial source for basic and scientific knowledge accumulation. In this set up, academia and scientific knowledge accumulation is considered mainly publicly funded and supported, so to simplify exposition I refer to it as a public research sector. Moreover, the analysis and results hold when academia generates both basic ideas and research tools (see Scotchmer, 2004, ch. 5).

[11] Cozzi (2007) proves that the standard Schumpeterian growth models are compatible with a positive and finite R&D investment by the incumbent monopolist. All the analysis of this paper is compatible with Cozzi's (2007) findings. Therefore, this model allows for a positive, yet non-strategic sighted, R&D investment by the incumbent monopolist.

the Schumpeterian creative destruction effect. The existence of a perfect stock market channels consumer savings to firms engaged in R&D. Throughout the analysis the number of R&D firms along any product line is assumed to be large enough so that each firm maximizes its expected profit flow.

# 4 Basic Framework

Let us assume continuous time and unbounded horizon. In this economy a mass $L_t > 0$ of infinitely lived families exists. Each family has an identical preference for non-negative consumption flows represented by the intertemporal utility function $\int_0^\infty e^{-rt} C_t dt$, where $C_t$ is the non negative consumption flow of each household, $r > 0$ is the common and constant subjective rate of time preference. The linear instantaneous utility implies constant real interest rate always equal to $r$. Moreover, each family is endowed with a unit mass of flow labor time. Population growth is constant and equal to $g_L > 0$. The labor market is perfect and the inelastic supply of labor $L_t$ is instantaneously employed by manufacturing firms and by the R&D sector.

Final output is produced by perfectly competitive firms combining a fixed factor with a large variety of intermediate goods, that is:

$$Y_t = M^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} di \tag{1}$$

with $0 < \alpha < 1$. $M$ is the aggregate mass of fixed factor (such as for example, "land, minerals, oils", etc.). $x_{it}$ is the amount of intermediate good $i$ produced and used as an input at a given time $t \geq 0$, and $A_{it}$ is the productivity parameter of the current version of that good. $N_t \in [0, \infty)$ denotes the mass of intermediate goods existing at date $t \geq 0$. The intermediate sector is characterized by free entry and exit, and by a constant returns to scale technology: workers can be hired by a continuum of firms that produce their intermediate goods on a one to one basis from labor. As patent protection is granted in each intermediate product line, instantaneous Bertrand competition guarantees that only the most advanced patent holder will be producing, and $N_t$ also denotes the mass of active intermediate good industries. The elasticity of substitution between intermediate products is equal to $\varepsilon \equiv \frac{1}{1-\alpha} > 1$.

The perfectly competing R&D firms try to achieve and appropriate the next generation of any intermediate good. According to Aghion and Howitt (1998), and Howitt (1999) the leading-edge technology is considered, with an economy-wide leading edge productivity parameter $A_t^{\max}$ that exerts positive R&D spillovers in all intermediate goods. When a new commercial valuable discovery is introduced along an intermediate product line $i$ - a better quality of that intermediate good is introduced - the productivity parameter $A_{it}$ in that line jumps to $A_t^{\max}$.

The technological frontier $A_t^{\max}$ grows at a rate proportional to the per product line rate of innovations. The Poisson arrival rate of innovation in any product line $i$ is $\lambda_A f(l_{At}, b_t)$. $\lambda_A$ is a productivity parameter, $l_{At} \equiv \frac{L_{At}}{N_t}$ is the industrial per product line research labor time, $b_t \equiv \frac{B_t}{N_t}$ is the per product line stock of basic knowledge. As remarked in the Introduction, Adams (2000), and Toole (1999) find that the stock of federally funded academic knowledge generates strong spillovers on industry, and that such spillovers are highly specific. This motivate us to adopt the per product line stock of basic knowledge spillovers specification, i.e. the variable $b_t$. Therefore, the function $f(l_{At}, b_t)$ captures the effect of different stages of a R&D process - basic, applied, and development - into generate a new marketable innovation (see Appendix A).

To simplify matter, as in Howitt (2000), the mass of intermediate goods is continuously enlarging thanks to serendipitous imitation. This allows the strong scale effect empirically rejected by Jones (1995) to be eliminated. In particular, the mass of intermediate products grows as a result of serendipitous imitation, not deliberate innovation. Any new product line draws a productivity level randomly from the invariant distribution $H(a)$ once introduced. Each person has the same propensity to imitate $\beta > 0$, thus the aggregate flow of new products is:

$$\dot{N}_t = \beta L_t \tag{2}$$

Since population grows at the constant rate $g_L$, the number of workers per product line $\frac{L_t}{N_t}$ converges monotonically to $\frac{g_L}{\beta}$.[12]

As an increasing number of intermediate goods is introduced in the economy, an innovation of a given size in any product line will have a smaller impact on the aggregate economy; hence the marginal impact of each innovation on the stock of public knowledge will be $\frac{\sigma}{N_t} A_t^{\max}$, where $\sigma > 1$ is a proportionality factor. The aggregate flow of vertical innovations is the number of intermediate goods $N_t$ times the expected flow of vertical innovations per industry line. The economy-wide rate of vertical technological progress is described by the following:

$$g_{At} \equiv \frac{\dot{A}_t^{\max}}{A_t^{\max}} = \frac{\sigma}{N_t} \int_0^{N_t} \lambda_A f(l_{At}, b_t)\, di = \sigma \lambda_A f(l_{At}, b_t) \tag{3}$$

Therefore, a new better quality version of any intermediate product is the result of the industrial effort that renders marketable and commercial valuable the offsprings of both academic and industrial R&D efforts. The generic specification of $f(l_{At}, b_t)$ allows the Poisson arrival rate of innovations to capture many aspects of both industrial and academic R&D efforts. To fix ideas, only a share of the stock of basic knowledge can be usefully developed, and/or the duplication argument can be applied to both industrial and academic research efforts.[13]

[12] The dilution solution to the strong scale effect is the best way to fit the empirical evidence as proven in Madsen (2008). Importantly, in such a framework, an expanding mass of varieties is introduced because of a positive population growth rate. Whenever $g_L = 0$ in such a framework, also $\beta = 0$. Moreover if $g_L < 0$, then $\beta < 0$ should be assumed.

[13] The per product line stock of basic ideas $b_t$ encompasses all discoveries that can be useful to industry in a wide meaning. It includes all basic ideas that help each economic agent to have new ideas, insights, developments, etc.

According to this framework, in equilibrium we will observe an ever-evolving intersectoral distribution of the absolute productivity parameters $A_{it}$, with values ranging from 0 to $A_t^{\max}$. Defining $a \equiv \frac{A_{it}}{A_t^{\max}}$, we can concentrate on the relative intersectoral distribution, that - as shown in Aghion and Howitt (1998, ch. 3) and in Howitt (1999) - converges to the unique stationary distribution of relative productivity parameters - $a$ - characterized by cumulative distribution function $H(a) = a^{\frac{1}{\sigma}}$, with $0 \leq a \leq 1$. Every time a better quality of an intermediate good is introduced into the economy, the absolute distribution will be re-scaled rightward because the technological process rises to $A_t^{\max}$.

## 4.1 Manufacturing, Asset Market, and Industrial R&D

As the final output is produced by perfectly competitive firms, and because of drastic innovations, the inverse demand for an intermediate product $i$ coincides with its marginal productivity.[14] Applying Aghion and Howitt's (1992, 1998) methods, the intermediate good $i$ production level that maximizes the monopolist profits at time $t$ is

$$x_{it} = M \left( \frac{\alpha^2 A_{it}}{w_t} \right)^{\frac{1}{1-\alpha}},$$

where, without any loss of generality, the fixed factor $M$ is normalized to one, i.e. $M = 1$. As the distribution of relative productivities is unchanging, the product lines are not classified by their index $i$ but by their relative productivity $a \equiv \frac{A_{it}}{A_t^{\max}}$. Defining the productivity-adjusted real wage as $\omega_t \equiv \frac{w_t}{A_t^{\max}}$, the instantaneous labor demand function for an intermediate product with relative productivity $a$ at date $t$ is rewritten as:

$$\tilde{x}_{it} \left( \frac{\omega_t}{a} \right) = \left( \frac{\alpha^2 a}{\omega_t} \right)^{\frac{1}{1-\alpha}} \tag{4}$$

where $\tilde{x}_{it} \left( \frac{\omega_t}{a} \right)$ is a labor demand function for the manufacturing firm. The labor force employed in the manufacturing sector negatively depends on the productivity-adjusted real wage.

The industrial and academic R&D efforts allows the technological frontier to be increased. The commercial value of an innovation is given by the firm's expected stock market value that monopolizes the commercialization of a new intermediate product. Let $V_t$ be the expected stock market value of a new intermediate product with the maximum productivity $A_t^{\max}$.

Any R&D firm $j$ targeting an intermediate product $i$ chooses its R&D intensity to maximize $V_t \lambda_A \psi(l_{Ajt}, b_t) - (1-s) w_t l_{Ajt} - (1-\phi) V_t$, where $l_{Ajt}$ is the labor time flow employed by a R&D firm $j$ at time $t$, $\lambda_A \psi(l_{Ajt}, b_t)$ is the Poisson arrival rate of innovation of any firm $j$, with $\psi(l_{Ajt}, b_t)$ having constant returns

[14] All the results of the model also hold with a non-drastic innovation assumption. The drastic specification allows the same modeling structure as in Aghion and Howitt (1998) to be used. This simplify the exposition.

to scale jointly in $l_{Ajt}$ and $b_t$, $s$ is the subsidy of the industrial research labor cost, $(1-\phi)V_t$ is the cost of licenses to use patented academic basic ideas. This cost is assumed to be a constant fraction $(1-\phi)$ of the expected stock market value of a new patented intermediate product $V_t$.[15] Notice that, the per product line basic stock knowledge is taken as given by each firm. The solution of such a maximization problem determines the optimal firm's laboratory size $l^*_{Ajt}$.

We will focus on the symmetric steady state, that is $x_{it} = x_t$, $l_{Ait} = l_{At}$, etc., for every intermediate product line $i$.[16] Because the R&D sector is characterized by free entry and exit, in equilibrium the following condition must hold for a successful R&D firm: $V_t\left[\lambda_A \psi\left(l^*_{Ajt}, b_t\right) + \phi - 1\right] - (1-s)\, w_t l^*_{Ajt} = 0$ (see Appendix B, point B1). This condition can be rewritten as:

$$w_t = \frac{\left[\lambda_A \psi\left(l^*_{Ajt}, b_t\right) + \phi - 1\right]}{(1-s)\, l^*_{Ajt}} V_t \tag{5}$$

Notice that, in eq. (5) the wage paid to a researcher in the industrial R&D laboratory $w_t$ moves in the same direction as the average Poisson arrival rate of innovation of the industrial R&D laboratory itself, i.e. as $\frac{\lambda_A \psi\left(l^*_{Ajt}, b_t\right)}{l^*_{Ajt}}$. By following the same steps as in Aghion and Howitt (1998, ch. 3, Appendix) from eq. (5) it follows:

$$\begin{aligned} \frac{(1-s)l^*_{Ajt}}{\left[\lambda_A \psi\left(l^*_{Ajt}, b_t\right) + \phi - 1\right]} w_t = V_t = A_t^{\max} \int_0^\infty e^{-(r+g_A/\sigma)\tau} \tilde{\pi}\left(\omega e^{g_A \tau}\right) d\tau = \\ = A_t^{\max} \int_0^\infty e^{-(r+g_A/\sigma)\tau} \frac{1-\alpha}{\alpha} \omega_t \tilde{x}(\omega)\, e^{-\frac{\alpha}{1-\alpha} g_A \tau} d\tau \end{aligned} \tag{6}$$

On the right hand side of eq. (6) the discount rate $(r + g_A/\sigma)$, and the profit flows $A_t^{\max}\tilde{\pi}\left(\omega e^{g_A \tau}\right)$ accruing to a successful innovator from date $t$ to infinity have been considered.

In such a framework, any individual could choose to start up with an own R&D firm rather than to be employed in an industrial R&D laboratory. The model admits the existence of start-up companies with a laboratory size composed of more than a single researcher. Moreover, it is proven that, in equilibrium, an individual does not find it profitable to spend her labor time in both academic research and industrial R&D at the same time (see Appendix B, points B2 and B3).

## 4.2 Academic Research

The empirical evidence shows that a higher wage is gained on average in industrial R&D laboratories than in non-profit/government R&D organizations such

[15] The qualitative results of the model hold whenever the cost to develop academic basic ideas and/or research tools represents a fraction of $V_t$, which is a very general assumption because it is compatible with many plausible microfoundations. Notice that the cost $(1-\phi)V_t$ could also represent academic research funding from industry. In that case all the propositions and results continue to hold but the patent policy implication does necessarily not. See Scotchmer (2004).

[16] As proven by Cozzi (2005), Howitt's (1999) model admits a continuum of symmetric balanced growth paths.

as academia.[17] In order such an empirical evidence to be met, and following Aghion *et al.* (2005), individuals are assumed to care about their creative control, and academia is assumed to guarantee such a creative control more than the industrial sector does. In particular, individuals heterogeneously evaluate their creative control according to a random variable $\theta \in \left(0, \bar{\theta}\right)$, with $\bar{\theta} > 1$, distributed across individuals according to a continuous cumulative distribution function (cdf) $G(\theta)$. The usual properties $G'(\theta) > 0$, $G(0) = 0$, and $G\left(\bar{\theta}\right) = 1$ apply. In this way, an individual that values her creative control $\theta$, will accept to work in academia whenever she earns at least a wage $\frac{w_t}{\theta}$. Indeed, an individual that highly values her creative control will accept a lower wage to maintain it. As the academia (and government) is a rational agent, it will pay exactly $\frac{w_t}{\theta}$ to an individual that values her creative control $\theta$. Yet, academia will not pay a wage higher than the marginal productivity of labor, which is pinned down by the labor market wage rate $w_t$. Therefore, any individual that values her creative control $\theta$, such that $\frac{w_t}{\theta} > w_t$, will not be employed in academia, and she will work in the private sector, either manufacturing or industrial R&D.[18]

Let us consider the individual choice to be employed either in the industrial sector or in academia. The private sector pays a flow wage rate $w_t$ to any individual. Therefore, an individual that values her creative control $\theta$ will be employed in the private sector whenever

$$w_t - \lambda_B(e)\frac{w_t}{\theta} \leq 0 \tag{7}$$

The first term in inequality (7) indicates the wage rate in both manufacturing and industrial R&D sectors. The second term in inequality (7) indicates the flow return to be employed in academic research. The variable $e$ represents the productivity-adjusted per product line government R&D expenditures in academia, $\lambda_B(e)$ is a decreasing function of such expenditures capturing the academic 'research environment quality'. To fix ideas, higher government R&D expenditures - that can be interpreted as a better quality of laboratories' equipment and research environment in academia - does not enter in the contractual wage rate of researchers. Yet, such R&D expenditures allows a better 'research environment' - where an individual can express her creative control - to be created.[19] This implies that a researcher would want a higher wage premium to give up her creative control whenever a better academic 'environment' exists. In particular, a researcher that values her creative control $\theta$ would want a wage pre-

[17]For the empirical evidence about these facts see D'Este and Patel (2007), Stern (2004), SEI (2004, 2006).

[18]Notice that, because of such a labor market structure in the economy, each individual always spend all her time in working, and there is not any leisure-labor choice for individuals. Therefore, the only argument in the utility function is consumption.

[19]Such government R&D expenditures can include laboratories' equipment, libraries, computer softwares, dataset, etc. This mean that such expenditures allows each researcher in academia to work in a better 'research environment'. Notice that in the analysis the final product is a non-durable good. This implies that public R&D expenditures $e$ also are non-durable goods. Such an assumption allows the model to be simplified, but it does not play any role in results and policy implications.

mium $w_t - \lambda_B(e)\frac{w_t}{\theta}$ to give up such a creative control.[20] In such a framework, what matters for an individual to give up her creative control is the effective rents $\lambda_B(e)\frac{w_t}{\theta}$, that is different from the contractual wage rate $\frac{w_t}{\theta}$. Therefore, higher government R&D expenditures allows a better academic research environment to be obtained, and such R&D expenditures allows a larger number of individuals to be attracted in academia.

Notice that, a higher average wage is gained in an industrial R&D laboratory than in academia because of a bonus type reward mechanism as the empirical evidence shows to be in practice. This depends on the assumption of academia as a rational agent whereby it will not pay a wage rate higher than $w_t$, i.e. higher than the marginal productivity of labor.

Let us denote $\theta_0$ the threshold value that satisfies (7) as equality: the researchers that value their creative control $\theta_0$ are indifferent between trying to improve the quality of any existing intermediate good, to be employed in academic research, and to be employed in the manufacturing sector. The no-arbitrage equation (7) determines such a threshold value:

$$\theta_0 = \lambda_B(e) \tag{8}$$

which is constant along the BGP. Each individual with $\theta \leq \theta_0$ will find it profitable to be employed either in industrial R&D laboratories or in manufacturing. Hence, $G(\theta_0)L_t$ individuals will choose to be employed in the private sector. Instead, the individuals with $\theta > \theta_0$, that is $[1 - G(\theta_0)]L_t$, will decide to be employed in academia, and they will earn a lower wage than in the private sector. Notice that, in equilibrium, labor is inelastically supplied in the economy and each individual gains a wage that reflects either her marginal productivity or her creative control evaluation.

Given this endogenous labor allocation choice, the basic knowledge in academia $B_t$ is accumulated according to the following dynamic law:

$$\dot{B}_t = \frac{1}{\lambda_B(e)}\left[\int_{\theta_0}^{\bar{\theta}} \chi(\theta)\, G'(\theta)\, d\theta\right] L_t = \frac{m(\theta_0)\, L_t}{\lambda_B(e)} \tag{9}$$

where the productivity in academic research is positively affected by public R&D expenditures through the term $\frac{1}{\lambda_B(e)}$. The function $\chi(\theta)$ is any non-decreasing and positive real-valued function of the creative control $\theta$, $m(\theta_0)L_t \equiv \left[\int_{\theta_0}^{\bar{\theta}} \chi(\theta)\, G'(\theta)\, d\theta\right] L_t$ is the expected conditioned cumulated academic research effort.[21] Eq. (9) implies that the stock of basic knowledge $B_t$ is accumulated at the same rate as the population growth rate $g_L$.

[20] Goolsbee (1998) finds that increases in expenditures for public R&D have a significant effect in raising average S&E wages. This effect is predicted in the model as will be shown below. Moreover, because of the raising cost of technically skilled workers used in private R&D laboratories, government funding tends to "crowd out" private R&D. The theoretical results and causal mechanisms of the model allows such empirical findings to be explained.

[21] Notice that the function $\chi(\theta)$ can be a constant function. This implies that the individual productivity in creating new basic ideas in academia is constant and independent on the creative control evaluation. This seems the most neutral hypothesis to work with.

Let us consider again eq. (7). This model admits different ways a government can finance the stages of a R&D process. On the one hand, an undifferentiated subsidy $s$ is paid to industrial laboratories that allows the research cost to be reduced, and the expected stock market value of an innovation to be increased, as evident from eq. (5). On the other hand, public R&D expenditures $e$ allows the productivity of academic researchers to be improved. Therefore, the subsidy and public expenditures are two alternative ways to finance different R&D activities, once given government proceeds allows a balanced government budget to hold in equilibrium. To better understand such a point, a balanced government budget constraint is introduced.

The government uses proceeds from both academic innovations and a lump-sum tax $\tau$ to finance academic research, and to subsidize industrial R&D. A balanced government budget in each instant of time requires (see Appendix C):

$$\begin{aligned} \left[\int_{\theta_0}^{\bar{\theta}} \frac{1}{\theta} G'(\theta)\, d\theta\right] w_t L_t + s w_t N_t l_{At} + E\left(A_t^{\max}\right) L_t = \\ = N_t d\left(1-\phi\right) V_t + \tau Y_t \end{aligned} \tag{10}$$

The left hand side of eq. (10) represents public outlays to finance both industrial and academic R&D efforts. $\left[\int_{\theta_0}^{\bar{\theta}} \frac{1}{\theta} G'(\theta)\, d\theta\right] w_t L_t \equiv \eta\left(\theta_0\right) w_t L_t$ are total outlays for the wages paid to researchers in academia, $s w_t N_t l_{At}$ are total outlays to subsidize industrial R&D effort, $E\left(A_t^{\max}\right) L_t$ indicates public expenditures aimed to increase productivity of academic researchers, and are proportional to the leading-edge productivity parameter $A_t^{\max}$, and to population size $L_t$. The right hand side of eq. (10) indicates the total proceeds of the public sector. The government appropriates a fraction $(1-\phi)$ of the expected stock market value of a marketable idea $V_t$ from both successful and unsuccessful R&D firms; $d$ indicates the per product line number of R&D firms; $\tau$ is a constant lump-sum tax rate that is proportional to leading-edge technology parameter $A_t^{\max}$ and to population.[22]

## 4.3 Labor Market Equilibrium

Each researcher endogenously decides to allocate her labor time either in the inventive or in the manufacturing activity. Plugging these results in the manufacturing/industrial R&D no-arbitrage condition (6), and solving the integral yields:

$$\frac{(1-s)\, l_{Ajt}^{*}}{\left[\lambda_A \psi\left(l_{Ajt}^{*}, b_t\right) + \phi - 1\right]} = \frac{\frac{1-\alpha}{\alpha} \tilde{x}(\omega)}{r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A} \tag{11}$$

[22] In this framework all per capita/per product line variables are proportional to the leading-edge productivity parameter $A_t^{\max}$. Along the BGP, per capita consumption grows over time at the same rate as per capita final output $Y_t/L_t$. In fact, the final output is also the unique and homogeneous consumption good of the economy. Moreover, linear preferences imply a constant path of per capita consumption over time, and the productivity-adjusted per capita consumption is constant over time.

Solving the above equation for $\tilde{x}(\omega)$, the labor force employed in the production of the top quality intermediate good is obtained:

$$\tilde{x}(\omega) = \frac{(1-s)\, l^{*}_{Ajt}}{\left[\lambda_A \psi\left(l^{*}_{Ajt}, b_t\right) + \phi - 1\right]} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A\right) \frac{\alpha}{1-\alpha} \tag{12}$$

from which, by inverting eq. (12), it is possible to determine the productivity-adjusted real wage $\omega_t$.

The labor market clearing condition for manufacturing and industrial innovation is:

$$G(\theta_0)\, L_t = N_t l_{At} + N_t \int_0^1 \tilde{x}(\omega/a)\, h(a)\, da = N_t l_{At} + \frac{N_t \tilde{x}(\omega)}{1 + \frac{\sigma}{1-\alpha}} \tag{13}$$

where $h(a)$ is the density function of the relative productivities' cumulative distribution function $H(a)$.

The labor market clearing condition for academic research is:

$$[1 - G(\theta_0)]\, L_t = L_{Bt} \tag{14}$$

From eq. (1), and reclassifying intermediate goods by their relative productivities, the aggregate GDP can be written as (see Aghion and Howitt 1998, ch. 3, and Howitt 1999):

$$\begin{gathered} Y_t = A_t^{\max} N_t \int_0^1 a \tilde{x}(\omega/a)^{\alpha}\, h(a)\, da = \\ = A_t^{\max} N_t \int_0^1 \frac{1}{\sigma} a^{\frac{1}{\sigma}} \left(\frac{\alpha^2 a}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}} da = \frac{N_t A_t^{\max} \left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} \end{gathered} \tag{15}$$

Notice that, in the light of both eq.s (15) and (1), the productivity-adjusted fixed factor rent is:

$$\frac{re}{A_t^{\max}} = (1-\alpha)\frac{Y_t}{M A_t^{\max}} = (1-\alpha)\frac{N_t \tilde{x}^{\alpha}(\omega)}{\left(1 + \frac{\sigma}{1-\alpha}\right)} = (1-\alpha)\frac{N_t \left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} \tag{16}$$

Therefore, the fixed factor rent increases in the number of intermediate goods, simply because they complement it in the production of the final good; and it decreases in the productivity-adjusted real wage.

# 5 General Equilibrium

The economy has a unique rational expectation equilibrium on which rational individuals instantaneously jump on. From now onward the time index is eliminated for the sake of notational simplicity.

Let us consider the law of motion of the basic knowledge (9), along the BGP it is obtained:

$$b \equiv \frac{B}{N} = \frac{m(\theta_0)}{\lambda_B(e)\beta} \tag{17}$$

The labor demand in eq. (12) for the top quality intermediate good becomes:

$$\tilde{x}(\omega) = \frac{(1-s)\, l_{Aj}^{*}}{\left[\lambda_A \psi\left(l_{Aj}^{*}, b\right) + \phi - 1\right]} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A\right) \frac{\alpha}{1-\alpha} \tag{18}$$

Let us consider both eq.s (18) and (13), along the rational expectation equilibrium, a positive and finite value for the per product line industrial research effort exists such that:

$$l_A = \left[\frac{L}{N} G(\theta_0) - \frac{\frac{(1-s) l_{Aj}^{*}}{\left[\lambda_A \psi\left(l_{Aj}^{*}, b\right) + \phi - 1\right]} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A\right) \frac{\alpha}{1-\alpha}}{\left(1 + \frac{\sigma}{1-\alpha}\right)}\right] \tag{19}$$

From eq. (19) the per product line industrial research effort along the BGP is obtained (see Appendix D). The per product line public R&D employment is:

$$l_B = \left[1 - G(\theta_0)\right] \frac{g_L}{\beta} \tag{20}$$

The per capita output is:

$$\frac{Y}{L} = \frac{\frac{N}{L} A^{\max} \tilde{x}^{\alpha}(\omega)}{\left(1 + \frac{\sigma}{1-\alpha}\right)} = \frac{\frac{\beta}{g_L} A^{\max} \left(\frac{\alpha^2}{\omega}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} \tag{21}$$

where eq. (15) has been used.[23] Therefore, the per capita output growth rate equals the technological frontier growth rate:

$$g_{Y/L} = g_A = \sigma \lambda_A f(l_A, b) \tag{22}$$

# 6 Comparative Statics

This section compares the effect of either a higher subsidy $s$ or a lower fixed cost $(1-\phi)$ to acquire licenses of patented academic ideas on the industrial research incentive, and on the per capita output growth rate of the economy. The last part of the section shows how public R&D expenditures $e$ and the population

[23] Notice that $Y7L = 0$ if $\beta = 0$. In such a framework the parameter $\beta$ is introduced because population size is not constant over time. In this way, the serendipitous imitation assumption allows the strong scale effect to be eliminated in models with growing population. Indeed, with a constant population size, the parameter $\beta$ would not appear in the model, so that $\beta = 0$ would necessarily be associated with $g_L = 0$. Yet, both the results and policy implications of such a framework are tied to the population growth rate, and therefore $\beta > 0$ always holds whenever $g_L > 0$.

growth rate - jointly with some structural parameters - contribute to explain the mixed empirical evidence about a substitutability/complementarity relationship between industrial and public research investments. At the same time, the effect of higher public R&D expenditures on growth is analyzed. In all the comparative statics analysis a change in public outlays is always offset by a change in the lump sum tax rate $\tau$ that allows a balanced government budget in eq. (10) to hold in equilibrium. In other words, a change either in the subsidy $s$ or in public R&D expenditures $e$ is offset by an appropriate change in the lump sum tax rate $\tau$ such that a balanced government budget in eq. (10) holds in each instant.

## 6.1 Subsidies and Licenses

To study the effect of either a higher subsidy $s$ of the industrial R&D cost or a lower industrial cost $(1-\phi)$ to use academic patented ideas, let us consider eq.s from (17) to (22). Along the rational expectation equilibrium, the following can be stated:

**Proposition 1** *Along the rational expectation BGP, a constant fraction of population is employed in manufacturing, academic and industrial R&D sectors. Along the BGP, an increase either in the parameter $\phi$ - i.e. a lower industrial cost $(1-\phi)$ to use academic ideas - or in the subsidy $s$ positively affects the per capita output growth rate, and negatively affects the per capita output level.*

***Proof.*** *See Appendixes D and E* ■

The above proposition states a substantial equivalence - as two alternative ways to spur industrial research investments - between a lower cost to use academic basic ideas, and a lower research labor cost due to a higher subsidy of the industrial R&D effort. Yet, some fundamental differences between such two policy instruments exist. Indeed, the cost to use patented academic ideas is tied to a patent policy design. In general, a patent policy 'design' concerns the uncertainty aspect of a R&D process and it involves the political, executive, jurisprudential authorities of a country. Therefore, this policy 'design' strongly shapes the institutional set-up in which both industrial and public R&D operate. The subsidy does not shape the institutional set-up of the economy as a patent policy 'design' does, and it concerns the certainty aspect of a R&D process.[24] Moreover, the subsidy can be also managed in short time horizon. By keeping in mind such deep differences, the effect of either a lower cost to use academic ideas or a higher subsidy $s$ is very different in magnitude, as the following states:

[24] Here I abstract from other institutional aspects. To fix ideas, subsidies are not immune from such distortions as measurement imperfections, lobbying, and moral hazard issues. Such a distortions could concern government expenditures, subsidies, and are tied to some imperfect or incomplete information environment that is not a topic of the paper.

**Proposition 2** *Whenever condition (E6) holds along the BGP with a public balanced budget - i.e.* $s \geq 2 - \phi - \lambda_A \psi \left( l^*_{Aj}, b \right)$ *- a higher subsidy would produce a higher per capita output growth rate and a lower per capita output level than that obtained from a lower cost to use academic ideas, i.e. a higher* $\phi$*.*

***Proof.*** *See Appendix E* ■

To better understand the arguments, let us consider the government balanced budget along the BGP:

$$\begin{aligned} &\tfrac{g_L}{\beta} \eta \left( \theta_0 \right) \omega + s \bar{l}_A \omega + e = \\ &= \tfrac{\omega (1-\phi)(1-s) l^*_{Aj}}{\left[ \lambda_A \psi \left( l^*_{Aj}, b \right) + \phi - 1 \right]} d + \tau \tfrac{\left( \frac{\alpha^2}{\omega} \right)^{\frac{\alpha}{1-\alpha}}}{\left( 1 + \frac{\sigma}{1-\alpha} \right)} \end{aligned} \tag{23}$$

where $e \equiv \frac{E(A_t^{\max}) L_t}{A_t^{\max} N_t}$ is constant along the BGP, and $\bar{l}_A$ is the per product line industrial labor time research effort.

From condition (E6), the higher $\phi$ the lower a threshold subsidy $s$ effective in increasing the per capita output growth rate. This means that the larger the industrial appropriation of the expected stock market value of an innovation, the lower the subsidy an economy needs to spur industrial R&D investments, and the growth rate of a country. Eq. (23) allows the tax rate $\tau$ such that $\bar{s} = 2 - \phi - \lambda_A \psi \left( l^*_{Aj}, b \right)$ to be obtained. Therefore, given the patent policy 'design' - i.e. for a given value $\phi$ - and given a subsidy $s$, it is possible to pin down the tax rate such that a marginal increase in the subsidy greatly spurs the industrial development effort, and the per capita output growth rate of an economy.

As $\phi$ is a measure of the strength and tightness of IPR granted to academic basic ideas, proposition 2 has a main policy implication. In particular, because IPR protection for academic ideas represents a cost for R&D firms, a 'softer' protection for such worth ideas would reduce the industrial R&D cost, and it would improve the growth performance of a country. Therefore, two different regimes of IPR should be provided for academic ideas and industrial innovations. In this way, a higher value of the parameter $\phi$ can be obtained, and therefore also a low subsidy of the industrial R&D cost greatly spurs the innovative effort, and the per capita output growth rate of a country. When patents are explicitly considered, the different regime should concern both patentability requirement and patent breadth.[25]

[25] From a patent-design point of view, this implies that a far larger patentability requirement and a lower patent breadth should be provided to basic ideas than to industrial innovations. In fact, a larger patentability requirement and a lower patent breadth for academic ideas would allow a private firm to reduce the cost for licenses, and to get a higher share of a new product's market value. The patentability requirement is a minimum innovation size required to receive a patent. A patent breadth's put restrictions on the products other firms can produce without a license. In particular, the leading breadth limits future innovators by specifying superior products that other firms can not produce (see O'Donoghue, 1998, and O'Donoghue and Zweimuller, 2004). Based on these definitions, the leading breadth could only concern a new marketable product, and it could not apply at a basic innovation that - by its own definition - does not have an immediate and specific market application. However, if the leading breadth also applies at a basic innovation, a lower leading breadth should be

## 6.2 Public R&D Expenditures

Let us now consider the effect of higher public R&D expenditures $e$ on the industrial development effort, and on the per capita output growth rate of a country. As remarked above, in this framework productivity-adjusted per product line public expenditures $e$ affect the productivity of researchers. The effects of higher public R&D expenditures are summarized in the following

**Proposition 3** *When population growth rate is high enough along the BGP, higher public R&D expenditures determine: a) a higher per product line industrial R&D effort; b) a higher per capita output growth rate; c) a lower per capita output level.*

***Proof.*** *See Appendix F* ■

When the public sector invests resources in public R&D a larger population fraction will choose to be employed in the academic research sector. Yet, this endogenous choice by individuals can either crowd out or complement the industrial research effort. The result depends on a threshold for the population growth rate which is pinned down by both structural and institutional parameters of the economy.

When the population growth rate is not high enough, an increase in public R&D expenditures crowds out both industrial research effort and manufacturing employment (see Appendix F). Indeed, when population growth rate is not high enough intermediate firms can employe a very low population fraction, and the instantaneous profit flow of an intermediate firm is strongly reduced. This in turn implies a lower industrial R&D effort due to a too low market demand for each intermediate good. In this case, the final effect on the per capita output growth rate depends on the relative strength of academic discoveries and of industrial development effort in the function $f\left(\bar{l}_A, b\right)$. Whenever the spillovers of academic ideas on the industrial sector are strong enough, higher public R&D expenditures increase the per capita output growth rate even if they crowd out both industrial research investment and manufacturing employment.

Instead, when population grows at a high enough pace, higher public R&D expenditures complement industrial R&D effort, and only crowds out manufacturing employment. In such a case, the positive effect of higher public expenditures on the per capita output growth rate is magnified from both higher academic and industrial research efforts. More importantly, a positive and strong enough population growth rate represents a fundamental prerequisite for higher R&D expenditures to better off the growth performance of a country. Therefore, any policy that affects both the fertility and the mortality rate greatly contributes to affect the R&D effectiveness on the growth rate of the economy. Moreover, such results contribute to explain the mixed substitutability/complementarity relationship between industrial and public R&D investments.

Notice that, an increase in the subsidy $s$ always positively affects the per capita growth rate of an economy, once a higher tax rate allows a government

granted at public basic innovations.

balanced budget to be obtained. Instead, such a positive effect can not always be obtained by an increase in public R&D expenditures $e$. As both a subsidy and public R&D expenditures represent outlays for a government, a trade-off between such different ways to finance R&D exists, for given government proceeds. In the analysis, if both $s$ and $e$ are increased, it is implicitly assumed that a change in $\tau$ allows a balanced government budget to hold in equilibrium. To fix ideas, a change in $s$ is not offset by a change in $e$ of opposite sign, but by a change in $\tau$. Therefore, an increase in both $s$ and $e$ implies a higher fiscal burden on consumers, and therefore a lower consumption level, and a lower utility. Moreover, when population growth rate is not high enough, an increase in both $s$ and $e$ generates an opposite effect on the per capita output growth rate of the economy. The final effect - either positive or negative - depends on structural parameters, on specific functional form in both manufacturing and R&D sectors - i.e. on technology - and on the net fertility rate of the economy.

# 7 Conclusions

In most developed economies the industrial sector invests large amount in R&D to gain or to maintain technological supremacy on the global marketplace. To this aim, firms incur a research fixed cost to introduce marketable innovations with a fundamental uncertainty for the offsprings of such effort. A traditional legal instrument intended to provide an economic incentive consists in granting IPR protection to industrial innovations. Moreover, public institutions of several OECD countries extended IPR protection to academic basic ideas - i.e. to new knowledge that does not have an immediate and specific market application - with the aim to enhance the transfer of knowledge from universities to industry, the commercialization of academic ideas, and the academic contribution to innovation and growth. The existence of strong links between academia and industry is also well documented, among others, by large and direct spillovers of university's basic ideas on industry.

This paper is a first theoretical attempt to shed light on the macroeconomic implications of such strong links within a dynamic general equilibrium framework. To this aim, a quality ladder endogenous growth model where the stages of a R&D process are explicitly separated is adopted. Technological frontier increases as a result of the research activity in both government funded academia and industry. As new and worth ideas at all stages of the R&D process are granted of a full IPR protection, the industry pays to use academic basic ideas. Moreover, industrial R&D firms obtain a subsidy for their research cost.

The results show that either a higher subsidy to industrial research effort or a lower cost paid by firms to use patented academic ideas, increase both the industrial innovative effort and the per capita output growth rate, while they reduce the per capita output level. Such results have a main policy implication. Once IPR are granted at all stages of a R&D process, a softer protection should be set for academic ideas than for industrial innovations. Indeed, in such a way the cost to use academic ideas is lower, and this spurs the industrial R&D effort.

Moreover, it is shown that higher public R&D expenditures can either crowd out or complement the industrial research investment. In particular, when population grows at a high enough pace - as compared with institutional and structural parameters - higher public R&D expenditures complement the industrial research effort, and both positively affect the per capita output growth rate of the economy. Such a result contributes to theoretically explain the mixed empirical evidence about a substitutability/complementarity relationship between public and industrial research investments.

## Appendix A

1. Any R&D firm $j$ has a Poisson arrival rate of innovation $\lambda_A \psi(l_{Ajt}, b_t)$ at a given time $t \geq 0$. The function $\psi(l_{Ajt}, b_t)$ has constant returns to scale jointly in $l_{Ajt}$ and $b_t$, with decreasing returns to scale in $l_{Ajt}$. Along the BGP, a symmetry in each product line $i$ between the R&D firms exists, so that $l_{Ajt} = \frac{l_{At}}{d}$, where $d$ is the number of R&D firms in the product line $i$. Because $\psi(l_{Ajt}, b_t)$ is homogeneous of degree one, it is $\psi(dl_{Ajt}, db_t) = d\psi(l_{Ajt}, b_t) \equiv f(l_{At}, b_t)$ The Poisson arrival rate of innovation is independently distributed across firms, across industry lines, and over time. Q.E.D.

The per product line stock of basic knowledge $b_t$ encompasses all academic ideas, either granted of IPR protection or not. The share of public discoveries granted of IPR protection - for which industrial R&D pays for their use - can be described as a continuous random variable $p \in [0, \bar{p}]$, with a common cumulative distribution function $F(p)$ and density $f(p)$ for all product lines $i \in [0, N_t]$. It is assumed that the average value of patented basic ideas is the same along each product line, i.e. $\tilde{p} = \int_0^{\bar{p}} p dF(p)$. Because there exists a continuum of intermediate product lines, the Strong Law of large Numbers implies that per product line share of patented public ideas acquired by the industrial sector is deterministic, i.e. $\frac{1}{N_t}\int_0^{N_t} p_i$ converges to $\tilde{p}$ with probability 1.

## Appendix B

1B. Each R&D chooses its laboratory size by maximizing:

$$V_t \lambda_A \psi(l_{Ajt}, b_t) - (1-s) w_t l_{Ajt} - (1-\phi) V_t,$$

the first order condition for a private R&D firm implies $V_t \lambda_A \psi_1(l^*_{Ajt}, b_t) \leq (1-s) w_t$, where $\psi_1(l^*_{Ajt}, b_t) = \frac{\partial \psi(l^*_{Ajt}, b_t)}{\partial l_{Ajt}}$. As $\psi(l_{Ajt}, b_t)$ is assumed concave in $l_{Ajt}$, the first order condition is also sufficient for a maximum. The interior solution for at least one firm implies $V_t \lambda_A \psi_1(l^*_{Ajt}, b_t) = (1-s) w_t$.

The R&D sector is characterized by free entry and exit, therefore the following condition must be satisfied for any R&D firm:

$$V_t \lambda_A \psi(l^*_{Ajt}, b_t) - (1-s) w_t l^*_{Ajt} - (1-\phi) V_t \geq 0.$$

The symmetry assumption for industrial R&D firms along each product line implies that the free entry condition can be rewritten as: $V_t \lambda_A \psi\left(\frac{l_{At}}{d}, b_t\right) - (1-s) w_t \frac{l_{At}}{d} - (1-\phi) V_t \geq 0$, where $d$ is the number of industrial R&D firms in a product line $i \in [0, N_t]$. When the marginal firm enters in the R&D race this condition will be binding, at that point the marginal firm will be indifferent between to enter or do not enter in the R&D race. This allows the number of industrial R&D laboratories $d$ in each product line to be pinned down.

Notice that, by combining the first order condition and the free entry condition the following inequality $\lambda_A \psi(l^*_{Ajt}, b_t) + \phi - 1 \geq \lambda_A l^*_{Ajt} \psi_1(l^*_{Ajt}, b_t)$ is obtained. This condition - that can be rewritten as $\frac{\lambda_A \psi(l^*_{Ajt}, b_t) + \phi - 1}{\lambda_A l^*_{Ajt}} \geq \psi_1(l^*_{Ajt}, b_t)$ - holds along the BGP equilibrium. In fact, because $\psi(l^*_{Ajt}, b_t)$ is concave in $l_{Ajt}$, the average productivity $\frac{\psi(l_{Ajt}, b_t)}{l_{Ajt}}$ is decreasing. Therefore, the condition

$\frac{\psi\left(l_{Ajt}^{*},b_{t}\right)}{l_{Ajt}^{*}} \geq \psi_{1}\left(l_{Ajt}^{*},b_{t}\right)$ is always satisfied. It is assumed that many firms can be ran in the economy, so that the inequality tend to be as close as possible to an equality.

2B. Let us turn to analyze the individual choice of either to start-up with an own research firm or to be employed in a research team within a R&D firm. The existence of industrial R&D firms employing more than one researcher is proven to hold along any BGP with a subsidy $s \in (0,1)$. A researcher that decides to start-up with an own R&D firm has expected returns $V_t \frac{\lambda_A \psi(l_{Ajt},b_t)}{(1-s)l_{Ajt}} - (1-\phi)V_t$. Indeed, the function $\psi(l_{Ajt},b_t)$ is homogeneous of degree one, and therefore $\frac{\lambda_A \psi(l_{Ajt},b_t)}{(1-s)l_{Ajt}} = \frac{\lambda_A \psi(1,b_t/l_{Ajt})}{(1-s)}$. The first term - $V_t \frac{\lambda_A \psi(l_{Ajt},b_t)}{(1-s)l_{Ajt}}$ - is the expected discounted stock market value of a new patent for a researcher, and $(1-\phi)V_t$ is the cost to acquire academic patented ideas. The return for being employed in a R&D firm is the wage rate $w_t$. Therefore, an individual will choose to be employed in a R&D firm whenever the following inequality is satisfied:

$$V_t \frac{\lambda_A \psi(l_{Ajt},b_t)}{(1-s)\,l_{Ajt}} - (1-\phi)V_t \leq w_t = V_t \frac{\lambda_A \psi_1(l_{Ajt},b_t)}{(1-s)} \tag{B1}$$

where in the second equality the first order condition for a private R&D firm is used. By simplifying condition (A1), the following is obtained:

$$\frac{\psi(l_{Ajt},b_t)}{l_{Ajt}} - \psi_1(l_{Ajt},b_t) \leq \frac{(1-\phi)(1-s)}{\lambda_A} \tag{B2}$$

whenever inequality (B2) holds any individual will find it profitable to be employed in an industrial R&D laboratory rather than to start up with a single R&D firm. On the contrary, whenever the weak inequality (B2) is reversed, each individual will find it profitable to start up with an own R&D firm, and each research laboratory will be composed of a single researcher. Indeed, condition (B2) implies the existence of industrial R&D laboratories composed of more than one researcher. Therefore, the model admits the existence of start-up companies with a R&D laboratory composed of more than an individual.

Notice that in the economy will be only active individual R&D laboratories when either a subsidy $s$ or an intellectual appropriation parameter $\phi$ would tend to unity. For $s \to 1$, the labor force is entirely engaged in industrial R&D activity because of a fully subsidized research cost. In particular, when a subsidy is close to $s \to 1$, a worker will find it more profitable to be self-employed as a research firm rather than to be employed in an industrial R&D firm. Indeed, for $s \to 1$, an individual that starts-up with an own R&D firm has an expected flow return $V_t \frac{\lambda_A \psi(l_{Ajt},b_t)}{(1-s)l_{Ajt}} - (1-\phi)V_t$, with the fixed cost $(1-\phi)V_t \to 0$ because $V_t \to 0$ for $s \to 1$. Instead a researcher employed in an industrial R&D laboratory has an expected flow return of $w_t = \frac{V_t \lambda_A \psi_1(l_{Ajt},b_t)}{(1-s)}$. Notice that, as $s \to 1$, the labor force demand in the R&D sector push up the wage rate $w_t$ to infinity, this implies that $V_t \lambda_A \psi_1(l_{Ajt},b_t)$ tends to zero at a lower rate than $(1-s)$ in the ratio $\frac{V_t \lambda_A \psi_1(l_{Ajt},b_t)}{(1-s)}$. Therefore, when considering the flow returns to be self-employed as a R&D firm - i.e. the ratio $\frac{V_t \lambda_A \psi(l_{Ajt},b_t)}{(1-s)l_{Ajt}}$ - also the term $\frac{V_t \lambda_A \psi(l_{Ajt},b_t)}{l_{Ajt}}$ tends to zero at a lower rate than $(1-s)$. Comparing the flow returns of a single R&D firm $\frac{V_t \lambda_A \psi(l_{Ajt},b_t)}{(1-s)l_{Ajt}}$ with that of a researcher employed in a R&D laboratory $\frac{V_t \lambda_A \psi_1(l_{Ajt},b_t)}{(1-s)} = w_t$, it is $\frac{V_t \lambda_A \psi(l_{Ajt},b_t)}{(1-s)l_{Ajt}} > \frac{V_t \lambda_A \psi_1(l_{Ajt},b_t)}{(1-s)}$ as $s \to 1$, because $\frac{\lambda_A \psi(l_{Ajt},b_t)}{l_{Ajt}} > \lambda_A \psi_1(l_{Ajt}, b_t)$. Therefore, as $s \to 1$ each individual will find it more profitable to run an individual R&D firm, and in the economy only R&D laboratories composed of no more than an individual will exist.

For $s \in (0,1)$, more than one researcher will be employed in each R&D firm, and therefore the above analysis applies along any BGP with a subsidy $s \in (0,1)$. This means that, whenever industrial R&D activity is positively but not fully subsidized, each industrial R&D firm employes more than one researcher, and each individual researcher earns the wage rate $w_t$. Q.E.D.

3B. This part formally proves that an individual does not find it profitable to spend her labor time in both academic research and industrial R&D at the same time. Suppose an individual spend a fraction $\upsilon$ of her labor time in academia earning $\frac{w_t}{\theta}$, and she spend the remaining fraction $(1-\upsilon)$ in the private sector earning $w_t$. In such a case the individual would strictly prefer to divide her labor time in both activity than in only the private sector whenever $\frac{w_t}{\theta}\upsilon + (1-\upsilon) w_t > w_t$. This is true whenever $\theta < 1$. Yet, in such a case the academia would not pay a wage $\frac{w_t}{\theta}$ because it is higher than $w_t$. In the case of a weak inequality $\frac{w_t}{\theta}\upsilon + (1-\upsilon) w_t \geq w_t$, it is $\theta \leq 1$, and the inequality should be rewritten as $w_t\upsilon + (1-\upsilon) w_t = w_t$. In such a case it is assumed that the individual will work in the private sector only. Instead an individual that only work in academia would prefer to spend her labor time in both academia and private sector whenever $\frac{w_t}{\theta}\upsilon + (1-\upsilon) w_t \geq \frac{w_t}{\theta}$. This is true whenever $\theta \geq 1$. Yet, in such a case the individual would prefer to maintain her creative control for all her labor time rather than to spend any positive fraction $(1-\upsilon) > 0$ in the private sector. Q.E.D.

## Appendix C

In this Appendix the eq. (10) is obtained. By following the same steps as in Aghion and Howitt (1998), the profit flow of a monopolistic firm that manufactures an intermediate product $i$ with productivity $A_{it}$ is

$$\pi_{it} = A_t^{\max} \frac{1-\alpha}{\alpha} \omega_t \left(\frac{\alpha^2}{\omega_t}\right)^{\frac{1}{1-\alpha}} a^{\frac{1}{1-\alpha}} = A_t^{\max} \tilde{\pi}(\omega) a^{\frac{1}{1-\alpha}}$$

where $\omega_t \equiv \frac{w_t}{A_t^{\max}}$ is the productivity-adjusted real wage, $\tilde{\pi}(\omega)$ is the profit flow of the intermediate good with the maximum productivity parameter $A_t^{\max}$. The expected stock market value of the last successful R&D firm that has productivity $A_t^{\max}$ is described by eq. (5) in the text. The expected stock market value of an intermediate product $i$ with absolute productivity $A_{it}$, and relative productivity $\frac{A_{it}}{A_t^{\max}}$ is $V_{it} = V_t a^{\frac{1}{1-\alpha}}$. Therefore, the cumulative expected stock market value of all manufacturing monopolies at a given time $t \geq 0$ is:

$$\int_0^{N_t} V_{it} di = N_t \int_0^1 V_{it} dH(a) = N_t V_t \int_0^1 a^{\frac{1}{1-\alpha}} dH(a) = \frac{N_t V_t}{1 + \frac{\sigma}{1-\alpha}} \tag{C1}$$

Let $d$ be the number of the per product line outsider R&D firms. The public sector appropriates a share $(1-\phi)$ of the expected stock market value of a patented idea from each R&D firm, successful and unsuccessful. Therefore a balanced budget requires

$$\begin{aligned} \eta(\theta_0) w_t L_t + s w_t N_t l_{At} + E\left(A_t^{\max}\right) L_t = \\ = d N_t (1-\phi) V_t + \tau Y_t, \end{aligned} \tag{C2}$$

In this setting the Arrow's effect is assumed to be at work. This implies that the incumbent firm does not find it profitable to undertake R&D. Yet, as proven by Cozzi (2007), this framework can not exclude a positive investment in R&D of the incumbent firm. If this argument would be considered, all the analysis remained valid by simply replacing $n = d + 1$ to $d$ in the paper. Q.E.D.

## Appendix D

In this Appendix the per product line industrial research labor time $l_{At}$ along the BGP is obtained. From now onward the time index is eliminated for the sake of notational simplicity. By eq. (19) the following is obtained:

$$\begin{aligned} \frac{\frac{L}{N} G(\theta_0)\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}\left[\lambda_A \psi\left(l_{Aj}^*, b\right)+\phi-1\right]}{(1-s) l_{Aj}^*} - r - \frac{\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}\left[\lambda_A \psi\left(l_{Aj}^*, b\right)+\phi-1\right]}{(1-s) l_{Aj}^*} l_A = \\ = \lambda_A \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) f(l_A, b) \end{aligned} \tag{D1}$$

where - along the BGP - $\frac{L}{N} = \frac{g_L}{\beta}$. Let us consider the left hand side of eq. (D1). The research labor time of a R&D firm $j$ is $l_{Aj}^* = \frac{l_A}{d}$. For $l_A \to 0$, so that also $l_{Aj}^* \to 0$, the left hand side of eq. (D1) tends to $+\infty$. Therefore the left hand side of eq. (D1) is always strictly positive for $l_{Aj}^* \to 0$. For $l_A \to \frac{g_L}{\beta} G(\theta_0)$, the left hand side of eq. (D1) tends to $\frac{\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}\left[\lambda_A \psi\left(\frac{g_L}{d\beta} G(\theta_0), b\right)+\phi-1\right]}{(1-s)\frac{g_L}{d\beta} G(\theta_0)} \left[\frac{g_L}{\beta} G(\theta_0) - l_A\right] - r \to -r < 0$.

Let us turn to prove that the left hand side of eq. (D1) is a strictly monotonic decreasing function of $l_{At}$. Let us consider eq. (19) rewritten as:

$$\begin{aligned} \Lambda = \frac{\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}\left[\lambda_A \psi\left(l_{Aj}^*, b\right)+\phi-1\right]}{(1-s) l_{Aj}^*} \left[\frac{L}{N} G(\theta_0) - l_A\right] - r + \\ -\lambda_A \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) f(l_A, b) \end{aligned} \tag{D2}$$

where the research labor time of a R&D firm $j$ is $l_{Aj}^* = \frac{l_A}{d}$. Therefore, it is

$$\begin{aligned} \frac{\partial \Lambda}{\partial l_A} = \frac{\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}\left[\frac{g_L}{\beta} G(\theta_0) - l_A\right]}{(1-s)} \frac{\left[\lambda_A \frac{l_{At}}{d} \psi_1\left(l_{Aj}^*, b\right) - \frac{1}{d}\left[\lambda_A \psi\left(l_{Aj}^*, b\right)+\phi-1\right]\right]}{\left[\frac{l_A}{d}\right]^2} + \\ - \frac{\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}\left[\lambda_A \psi\left(l_{Aj}^*, b\right)+\phi-1\right]}{\frac{(1-s) l_A}{d}} - \lambda_A \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) f_1(l_A, b) < 0 \end{aligned} \tag{D3}$$

The first order condition for a maximum profit and the free entry condition in the R&D race imply $\frac{\lambda_A \psi\left(l^*_{Ajt}, b_t\right)+\phi-1}{\lambda_A l^*_{Ajt}} \geq \psi_1\left(l^*_{Ajt}, b_t\right)$, and therefore it also is $\left[\lambda_A \frac{l_{At}}{d}\psi_1\left(l^*_{Aj}, b\right) - \frac{1}{d}\left[\lambda_A \psi\left(l^*_{Aj}, b\right)+\phi-1\right]\right] \leq 0$. Moreover, the labor market clearing condition implies $\frac{g_L}{\beta} G\left(\theta_0\right) \geq l_A$. Therefore, inequality (D3) is proven. The left hand side of eq. (D1) is a strictly monotonic decreasing function of $l_A \in \left[0, \frac{g_L}{\beta} G\left(\theta_0\right)\right]$, with values ranging from $+\infty$ to $-r$.

The right hand side of eq. (D1) is assumed to be an increasing and concave function of $l_A$, with $b$ taken as given by each R&D firm, and with $f\left(0,b\right) < +\infty$. From the above conditions, the per product line industrial research labor time effort $0 < \bar{l}_A < \frac{g_L}{\beta} G\left(\theta_0\right)$ along the balanced growth path is uniquely determined. Q.E.D.

## Appendix E

In the first part of this Appendix the effect of a lower cost for academic patented ideas - i.e. a higher value of the parameter $\phi$ - on the economic performance of a country is obtained; $\phi \in (0,1)$ is assumed. In the second part the effect of a higher R&D subsidy is analyzed. In the last part a comparison between the first two points is made.

1E. Let us consider eq. (D2). By using the Implicit Function Theorem it is obtained:

$$\frac{\partial l_A}{\partial \phi} = -\frac{\frac{\partial \Lambda}{\partial \phi}}{\frac{\partial \Lambda}{\partial l_A}} = -\frac{\frac{\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}}{(1-s)l^*_{Aj}}\left[\frac{g_L}{\beta} G\left(\theta_0\right) - \bar{l}_A\right]}{\frac{\partial \Lambda}{\partial l_A}} > 0 \qquad \text{(E1)}$$

where $\frac{\partial \Lambda}{\partial l_A} < 0$ from inequality (D3), and $\left[\frac{g_L}{\beta} G\left(\theta_0\right) - \bar{l}_A\right] > 0$. Therefore, along the BGP, a higher intellectual appropriation parameter $\phi$ increases the per product line industrial R&D effort. Q.E.D.

In order to determine how a lower cost for academic patented ideas - i.e. a higher $\phi$ - affects the market demand for any existing intermediate good, the labor market clearing condition is used:

$$L = G\left(\theta_0\right) L + \left[1 - G\left(\theta_0\right)\right] L = N\bar{l}_A + \frac{N\tilde{x}\left(\omega\right)}{1+\frac{\sigma}{1-\alpha}} + Nl_B \qquad \text{(E2)}$$

where $l_B = \frac{L_B}{N} = \left[1 - G\left(\theta_0\right)\right]\frac{L}{N}$ denotes the per product line basic research effort. From eq. (7) a constant threshold ability parameter $\theta_0$ is obtained. Therefore - along the new BGP with a higher value of parameter $\phi$ - the per product line basic research effort $\left[1 - G\left(\theta_0\right)\right]\frac{L}{N}$ is constant and equal to $\left[1 - G\left(\theta_0\right)\right]\frac{g_L}{\beta}$. Moreover, eq. (E1) proves that, along the new BGP, the per product line vertical research effort is higher. Therefore, eq. (E2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher value of parameter $\phi$ determines a lower per capita output level. Q.E.D.

The positive effect of a change in the parameter $\phi$ on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial \phi} = \sigma \lambda_A \frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A} \frac{\partial \bar{l}_A}{\partial \phi} > 0 \tag{E3}$$

where the inequality follows from eq. (E1). Q.E.D.

2E. This part analyses the effect of a change in the subsidy $s$ on the economic performance of the economy; $s \in (0, 1)$ is assumed. From eq. (D2), and by using the Implicit Function Theorem the following is obtained:

$$\frac{\partial l_A}{\partial s} = -\frac{\frac{\partial \Lambda}{\partial s}}{\frac{\partial \Lambda}{\partial l_A}} = -\frac{\frac{l_{Aj}^{*}\left(1+\frac{\sigma}{1-\alpha}\right)\frac{1-\alpha}{\alpha}\left[\lambda_A \psi\left(l_{Aj}^{*}, b\right)+\phi-1\right]}{\left[(1-s) l_{Aj}^{*}\right]^2}\left[\frac{g_L}{\beta} G\left(\theta_0\right)-\bar{l}_A\right]}{\frac{\partial \Lambda}{\partial l_A}} > 0 \tag{E4}$$

where $\frac{\partial \Lambda}{\partial l_A} < 0$ from inequality (D3), and $\left[\frac{g_L}{\beta} G\left(\theta_0\right) - \bar{l}_A\right] > 0$. Therefore, along the BGP a positive relationship between the subsidy $s$ and the per product line industrial R&D labor time $\bar{l}_A$ is proven. Q.E.D.

In order to determine the effect of a higher subsidy $s$ on the market demand for any existing intermediate good, the labor market clearing condition (E2) is used. As proven above, along the BGP, the per product line basic research effort is constant and equal to $\left[1 - G\left(\theta_0\right)\right] \frac{g_L}{\beta}$. Moreover, eq. (E4) proves that - along the new BGP with a higher subsidy to R&D firms - the per product line vertical research effort is higher. Therefore, eq. (E2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher subsidy $s$ determines a lower per capita output level. Q.E.D.

The positive effect of a change in the subsidy to private research effort $s$ on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial s} = \sigma \lambda_A \frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A} \frac{\partial \bar{l}_A}{\partial s} > 0 \tag{E5}$$

where the inequality follows from eq. (E4). Q.E.D.

3E. This part compares the effect of a marginal change in the appropriation parameter $\phi$ with the effect of a marginal change in the subsidy $s$. Along a new BGP with a larger value of either $\phi$ or $s$ both a higher per product line private innovation effort and a higher per capita output growth rate are obtained. In order to compare the magnitude of these effects it suffices to consider eq.s (E1) and (E4). Whenever the following condition is satisfied $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$, an increase in the subsidy generates the same economic effects as an increase in the appropriation parameter $\phi$, but the former are higher in magnitude. In fact, from eq.s (E1) and (E4), it immediately follows that $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$ if and only if

$$s \geq 1 - \left[\lambda_A \psi\left(l_{Aj}^{*}, b\right) + \phi - 1\right] = 2 - \phi - \lambda_A \psi\left(l_{Aj}^{*}, b\right) \tag{E6}$$

Q.E.D.

Appendix F

In this Appendix the relationship between the per product line vertical R&D effort and the per product line stock of public basic knowledge is derived. From eq. (D2) and by using the Implicit Function Theorem it is obtained:

$$\frac{\partial l_A}{\partial b} = -\frac{\frac{\partial \Lambda}{\partial b}}{\frac{\partial \Lambda}{\partial l_A}} = -\frac{\frac{\left(1+\frac{\sigma}{1-\alpha}\right)\lambda_A \psi_2\left(l_{Aj}^*,b\right)\frac{1-\alpha}{\alpha}\left[\frac{g_L}{\beta}G(\theta_0)-\bar{l}_A\right]}{(1-s)l_{Aj}^*} - \lambda_A\left(1+\frac{\sigma\alpha}{1-\alpha}\right) f_2\left(\bar{l}_A, b\right)}{\frac{\partial \Lambda}{\partial l_A}} \tag{F1}$$

with $\psi_2\left(l_{Aj}^*, b\right) > 0$, $f_2\left(\bar{l}_A, b\right) > 0$, $\frac{\partial \Lambda}{\partial l_A} < 0$. The inequality (F1) will be strictly positive if and only if

$$\begin{aligned} g_L > \Bigg\{ & \lambda_A\left(1+\frac{\sigma\alpha}{1-\alpha}\right) f_2\left(\bar{l}_A, \frac{m(\lambda_B(e))}{\lambda_B(e)\beta}\right) * \\ & * \frac{(1-s)l_{Aj}^*}{\left(1+\frac{\sigma}{1-\alpha}\right)\lambda_A \psi_2\left(l_{Aj}^*, \frac{m(\lambda_B(e))}{\lambda_B(e)\beta}\right)\frac{1-\alpha}{\alpha}} + \bar{l}_A \Bigg\} * \frac{\beta}{G(\lambda_B(e))} \end{aligned} \tag{F2}$$

where - along the BGP - the per product line private research effort $\bar{l}_A$ is strictly lower than $\frac{g_L}{\beta} G\left(\lambda_B(e)\right)$. When inequality (F2) holds a complementarity relationship between public and private R&D effort is found.

By explicitly considering that productivity of public basic research positively depends on per product line public expenditures $e$, i.e. $\frac{\partial \lambda_B(e)}{\partial e} < 0$, the following inequality holds along the BGP: $\frac{\partial \theta_0}{\partial e} < 0$, because $\theta_0 = \lambda_B(e)$. In this case the calculation in eq. (F1) modifies to:

$$\begin{aligned} \frac{\partial l_A}{\partial e} = -\Bigg\{ & k\psi_2\left(l_{Aj}^*, b\right)\frac{\partial b}{\partial e}\left[\frac{g_L}{\beta}G(\theta_0) - \bar{l}_A\right] + k\psi\left(l_{Aj}^*, b\right)\frac{g_L}{\beta}\frac{\partial G(\theta_0)}{\partial e} + \\ & -\lambda_A\left(1+\frac{\sigma\alpha}{1-\alpha}\right) f_2\left(l_A, b\right)\frac{\partial b}{\partial e}\Bigg\} * \\ & * \left\{\frac{\partial \Lambda}{\partial l_A}\right\}^{-1} \end{aligned} \tag{F3}$$

where $k \equiv \frac{\left(1+\frac{\sigma}{1-\alpha}\right)\lambda_A \frac{1-\alpha}{\alpha}}{(1-s)l_{Aj}^*}$, $\frac{\partial b}{\partial e} > 0$ because along the BGP $b = \frac{m(\lambda_B(e))}{\lambda_B(e)\beta}$, and $m\left[\lambda_B(e)\right]$ is an increasing function of $e$; $\frac{\partial G(\theta_0)}{\partial e} < 0$. Condition (F3) is strictly positive whenever the following inequality for the population growth rate is satisfied:

$$\begin{aligned} g_L > \beta \Bigg\{ & k\psi_2\left(l_{Aj}^*, b\right)\bar{l}_A\frac{\partial b}{\partial e} + \lambda_A\left(1+\frac{\sigma\alpha}{1-\alpha}\right) f_2\left(l_A, b\right)\frac{\partial b}{\partial e} * \\ & * \left[k\psi_2\left(l_{Aj}^*, b\right)\frac{\partial b}{\partial e}G(\theta_0) + k\psi\left(l_{Aj}^*, b\right)\frac{\partial G(\theta_0)}{\partial e}\right]^{-1}\Bigg\} \end{aligned} \tag{F4}$$

Finally, whenever $g_L > \max\left\{(F2), (F4)\right\}$ a higher per product line basic knowledge - either exogenous or endogenously obtained through higher public expenditures - complements the industrial R&D effort, i.e. $l_A$ is higher. Q.E.D.

To determine the effects of a higher productivity of the basic research programs on the per capita output level, the labor market clearing condition (D2) is used:

$$L = G(\theta_0(e))L + 1 - G(\theta_0(e))L = N\bar{l}_A + \frac{N\tilde{x}(\omega)}{1+\frac{\sigma}{1-\alpha}} + Nl_B$$

where $l_B = \frac{L_B}{N} = [1 - G(\theta_0)]\frac{L}{N}$ is higher because $\frac{\partial G(\theta_0)}{\partial\theta_0}\frac{\partial\theta_0}{\partial e} < 0$, and $G(\theta_0)L$ is lower because $\frac{\partial G(\theta_0)}{\partial\theta_0}\frac{\partial\theta_0}{\partial e} < 0$. Moreover, eq. (F3) proves that, along the new BGP, the per product line vertical research effort is higher. Eq. (D2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. From eq. (21), it immediately follows that a higher productivity of basic research programs reduces the per capita output level. Finally, because the population employed by private firms is lower - $G(\theta_0)L$ is lower - and the per product line vertical research effort is higher, the negative effect on per capita output level is magnified by higher public R&D expenditures. Q.E.D.

The effect of a change in $e$ on the per capita output growth rate is:

$$\frac{\partial g_{Y/L}}{\partial e} = \sigma\lambda_A\left[\frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A}\frac{\partial\bar{l}_A}{\partial b} + \frac{\partial f\left(\bar{l}_A, b\right)}{\partial b}\right]\frac{\partial b}{\partial\lambda_B(e)}\frac{\partial\lambda_B(e)}{\partial e} \tag{F5}$$

When condition (F4) holds, eq. (F5) is strictly positive. Q.E.D.

When the population growth rate is not high enough, inequality (F3) is strictly negative and an increase in public expenditures to finance basic R&D reduces the industrial innovation effort. As eq. (F5) shows, the effect on the per capita output growth rate can not be univocally determined. It depends on the relative strength of partial derivative $\frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A}$ and $\frac{\partial f\left(\bar{l}_A, b\right)}{\partial b}$. Yet, the per capita output level is lower because of the lower market demand along each intermediate product line. Q.E.D.

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