

Non-Cooperative Game Theory: Bayesian Equilibrium

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Modeling incomplete information

- from incomplete to imperfect information
 - ✓ Nature's move: types and beliefs
- Bayesian equilibrium
- mechanism design
 - ✓ Revelation Principle
 - ✓ non-linear pricing and bilateral trade
- presentations:
 - ✓ multiproduct nonlinear pricing
 - ✓ public goods provision

What you don't know

- incomplete information
 - ✓ production cost in competition
 - ✓ evaluation of goods in trade, auction, public goods
 - ✓ case quality (damage, winning probability) in settlement bargaining
- Harsanyi's Nobel prize
 - ✓ from incomplete to imperfect information
 - ✓ Nature's move only reveals to the chosen one \Rightarrow type
 - ✓ ignorant others have beliefs
- common prior
 - ✓ if not, build into types (FT 6.3)

Multiple selves, each for themselves

- Bayesian Nash equil.
 - ✓ stra.: from types to actions
 - ✓ def. and computation: MWG DEF. 8.E.1 & PROP. 8.E.1
 - ✓ all types act for their own interests
 - ✓ optimal response to others' strategies, given belief
- MWG EX. 8.E.2: free-riding in R&D
 - ✓ cost of technology $c \in (0, 1)$
 - ✓ private value of technology θ_i^2 , where $\theta_i \sim U[0, 1]$

What we want

- economy: MWG 23.B
 - ✓ I agents, one principle (mechanism designer)
 - ✓ collective choice $x \in X$
 - ✓ types $\theta_i \in \Theta_i$, common prior $\phi(\theta)$
 - ✓ payoff $u_i(x, \theta_i)$
 - ✓ complete or incomplete information among agents
- social choice function $f : \Theta \rightarrow X$
 - ✓ public economics: income taxation, public goods
 - ✓ IO: trade, price discrimination, auction
 - ✓ law and economics: settlement

How we do it

- mechanism $\{S, g\}$ such that $g : S_1 \times \dots \times S_I \rightarrow X$
 - ✓ S_i : stra. set
 - ✓ g : outcome function
 - ✓ game: $\{S, g, \Theta, u, \phi\}$
- implementation: MWG DEF. 23.B.4
 - ✓ \exists one equilibrium stra. $\{s_i^*(\theta_i)\}_i$ such that $\forall \theta \in \Theta$,
 $g(\{s_i^*(\theta_i)\}) = f(\theta)$
 - ✓ what *type* of equil.? dominant stra. or Bayesian
- but, there are so many possible games!

Anything is doable if people are honest

- direct revelation mech.: $S_i = \Theta_i$ and $g(\theta) = f(\theta)$
- truthful implementation: telling truth is an equil.
 - ✓ $s_i^*(\theta_i) = \theta_i$

PROPOSITION (MWG 23.C.1&D.1)

(Revelation Principle) Any (S, g) implements $f \Rightarrow$ so must be $\tilde{S}_i = \Theta_i +$ truth-telling.

And this worths a Nobel prize

- implementable f
 - ✓ consider only direct revelation game
 - ✓ agents have to tell the truth: (IC)
 - ✓ agents are willing to play: (IR)
- optimal implementable f
 - ✓ optimal at what time? MWG DEF. 23.F.1-3
 - ✓ depending on information revelation
 - ✓ *ex post* efficiency

Examples

- discrete types: nonlinear pricing
 - ✓ FT 7.1.1
- continuous types: bilateral trade
 - ✓ Myerson-Satterthwaite Thm. (MWG PROP. 23.E.1)

Presentations

multiproduct nonlinear pricing

public goods provision